more spread out; the middle value, $100 million, is slightly less likely, but the extreme values are slightly more likely.

It’s this notion of the spread of a random variable that the variance is intended to capture. We measure the spread in terms of the squared deviation of the random variable from its mean. In case 1, the variance is therefore

\[ \text{case 1 variance} = 0.98(\text{OM} - \mu)^2 + 0.02(\text{OM} - \mu)^2 = 196M^2; \]

in case 2 it is

\[ \text{case 2 variance} = 0.9801(\text{OM} - \mu)^2 + 0.0198(\text{OM} - \mu)^2 + 0.0001(200M - \mu)^2 = 198M^2. \]

As we expected, the latter variance is slightly larger, because the distribution of case 2 is slightly more spread out.

When we work with variances, everything is squared, so the numbers can get pretty big. (The factor \( M^2 \) is one trillion, which is somewhat imposing even for high-stakes gamblers.) To convert the numbers back to the more meaningful original scale, we often take the square root of the variance. The resulting number is called the standard deviation, and it is usually denoted by the Greek letter \( \sigma \):

\[ \sigma = \sqrt{\text{variance}}. \]

The standard deviations of the random variables \( X_1 + X_2 \) in our two lottery strategies are \( \sqrt{196M^2} = 14.00M \) and \( \sqrt{198M^2} \approx 14.071247M \). In some sense the second alternative is about $71,247 riskier.

How does the variance help us choose a strategy? It’s not clear. The strategy with higher variance is a little riskier; but do we get the most for our money by taking more risks or by playing it safe? Suppose we had the chance to buy 100 tickets instead of only two. Then we could have a guaranteed victory in a single lottery (and the variance would be zero); or we could gamble on a hundred different lotteries, with a \( 0.99^{100} \approx 0.366 \) chance of winning nothing but also with a \textbf{nonzero} probability of winning up to $10,000,000,000. To decide between these alternatives is beyond the scope of this book; all we can do here is explain how to do the calculations.

In fact, there is a simpler way to calculate the variance, instead of using the definition (8.13). (We suspect that there must be something going on in the mathematics behind the scenes, because the variances in the lottery example magically came out to be integer multiples of \( M^2 \).) We have

\[
E((X - EX)^2) = E(X^2 - 2EX) + (EX)^2 \\
= E(X^2) - 2(EX)(EX) + (EX)^2,
\]

Another way to reduce risk might be to bribe the lottery officials. I guess that’s where probability becomes indiscreet. (N.B.: Opinions expressed in these margins do not necessarily represent the opinions of the management.)