we do not know the exact value represented by $U_{t-1}^i$, so we assume that it is exactly $U_{\text{max}}$. Very rarely, this will lead us to underestimate $U_t^i$, but we set $U_{\text{max}}$ to be large enough to avoid this problem in most cases.

Let $U_t = (U_t^i)_{i=1}^N$ be the vector collecting the numbers of units of each type that exist at time $t$. We refer to $U_t$ as the “battle space” at time $t$.

### 2.2 Observation Model

The observation model specifies the likelihood of an observation given a particular assignment to $U_t$. We define an observation as the number of units of each type that we saw during a time interval, $O_t = (O_t^i)_{i=1}^N$. We assume that we can distinguish between individual enemy units within an epoch, but not between epochs. That is, if we make two observations of a unit of a certain type during the same epoch, we know whether we have seen the same unit twice, or two different units. We can therefore model these observations as sampling without replacement from $U_t$ within an epoch. For observations in different epochs, we assume that we cannot tell whether a unit observed in one epoch is the same as a unit observed in a previous epoch.

If we observe an enemy unit, we know that the unit exists. If, on the other hand, we do not observe a unit, there are two possible explanations. Either the unit does not exist, or it does exist but we did not look hard enough for it. Hence our observation model needs to incorporate some measure of scouting “effort”. If we put little effort into scouting, failure to observe a unit does not tell us much about whether it exists. If we scout extensively and still do not see the unit, it probably does not exist.

We can exploit domain knowledge to come up with a measure of effort. In Starcraft, players construct most of their buildings in either their main base or their natural expansion. The main base is the area of the map where the player’s starting units appear at the beginning of the game, while the natural expansion is the location where it is most “natural” to construct a second base. Because buildings must be defended, it is tactically advantageous to keep them close together. In the early game, the main base and the natural expansion are thus the most important areas to scout, since that is where the buildings will be located. A natural measure of scouting effort, then, is the proportion of these two areas that have been seen. We denote this proportion for slice $t$ as $E_t$.

We must now decide how our scouting effort influences the number of units we observe. Our initial approach was to treat each observable unit as an independent Bernoulli trial, with probability of success $E_t$. An observation would then be a vector of Binomial random variables, $O_t^i \sim \text{Binomial}(U_t^i, E_t)$.

The Binomial model assumes that the locations of units are distributed uniformly and independently in space. However, this assumption is wrong. Units tend to cluster together. For example, the primary task of “worker” units is to gather resources, which they do by traveling back and forth between the city center and the resources. Thus, almost all worker units will be found in the area between the city center and the resources. If we see one worker, it is probably because we have seen part of this area, and we would expect to have seen most of the other workers, too. There is thus more variance in the observations than the Binomial model would predict; the data are overdispersed with respect to the Binomial distribution.

We account for overdispersion by placing a Beta prior on the success probability parameter of the Binomial, forming a Beta-Binomial model (Haseman and Kupper, 1979):

$$O_t^i \sim \text{BetaBinomial}(U_t^i, \mu_t^i, \rho_t^i) = \frac{B(O_t^i + \alpha_t^i, U_t^i - O_t^i + \beta_t^i)}{B(\alpha_t^i, \beta_t^i)}$$

where $B(x, y)$ is the beta function, and

$$\alpha_t^i = \mu_t^i \frac{1 - \rho_t^i}{\rho_t^i}, \quad \beta_t^i = (1 - \mu_t^i) \frac{1 - \rho_t^i}{\rho_t^i}.$$

We adopt the $(\mu, \rho)$ parameterization of the Beta distribution, where $\mu \in [0, 1]$ is the mean of the Beta and $\rho \in [0, 1]$ is the dispersion parameter, which can be thought of as the correlation between individual successes. The Beta distribution is a conjugate prior of the Binomial. When $\rho \to 0$, the Beta-Binomial approaches the Binomial, while as $\rho$ increases, the density spreads out and eventually becomes bimodal. The bi-modality captures the “clumpiness” of the units: depending on whether the part of the area of interest we saw contains the units, the success probability is either high or low, but probably not in the middle.

For each unit type $i$, we learn a mapping $f^i(E_t)$ from the observation effort to the mean and dispersion of a Beta-Binomial distribution:

$$\logit(\hat{\mu}_t^i) = a_0^i + a_1^i E_t$$
$$\logit(\hat{\rho}_t^i) = b^i.$$

This is then plugged into the Beta-Binomial to compute the likelihood of observing $O_t^i$ given that $U_t^i$ units exist. We learn a different mapping for each unit type, to allow for differences in dispersion and ease of observability between unit types. The regression coefficients are assumed constant in time, but $\hat{\mu}_t^i$ varies in time due to its dependence on $E_t$.