As an alternative to Figure 7.5.2a one can also plot $x_1$ or $x_2$ as a function of $t$; some typical plots of $x_1$ versus $t$ are shown in Figure 7.5.2b, and those of $x_2$ versus $t$ are similar. For certain initial conditions it follows that $c_1 = 0$ in Eq. (13), so that $x_1 = c_2 e^{-t}$ and $x_1 \to 0$ as $t \to \infty$. One such graph is shown in Figure 7.5.2b, corresponding to a trajectory that approaches the origin in Figure 7.5.2a. For most initial conditions, however, $c_1 \neq 0$ and $x_1$ is given by $x_1 = c_1 e^{3t} + c_2 e^{-t}$. Then the presence of the positive exponential term causes $x_1$ to grow exponentially in magnitude as $t$ increases. Several graphs of this type are shown in Figure 7.5.2b, corresponding to trajectories that depart from the neighborhood of the origin in Figure 7.5.2a. It is important to understand the relation between parts (a) and (b) of Figure 7.5.2 and other similar figures that appear later, since one may want to visualize solutions either in the $x_1$-$x_2$ plane or as functions of the independent variable $t$.

Consider the system

$$x' = \begin{pmatrix} -3 & \sqrt{2} \\ \sqrt{2} & -2 \end{pmatrix}x.$$  \hfill (14)

Draw a direction field for this system; then find its general solution and plot several trajectories in the phase plane.

The direction field for the system (14) in Figure 7.5.3 shows clearly that all solutions approach the origin. To find the solutions assume that $x = \xi e^{rt}$; then we obtain the algebraic system

$$\begin{pmatrix} -3 - r & \sqrt{2} \\ \sqrt{2} & -2 - r \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$  \hfill (15)