Equation (34) can be simplified further if the coefficient matrix \( P(t) \) is a constant matrix (see Problem 17).

Use the method of variation of parameters to find the general solution of the system

\[
x' = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} x + \begin{pmatrix} 2e^{-t} \\ 3t \end{pmatrix} = Ax + g(t).
\] (35)

This is the same system of equations as in Examples 1 and 2.

The general solution of the corresponding homogeneous system was given in Eq. (10). Thus

\[
\begin{pmatrix} e^{-3t} \\ -e^{-3t} \\ -e^{-t} \end{pmatrix}
\]

is a fundamental matrix. Then the solution \( x \) of Eq. (35) is given by \( x = \Psi(t)u(t) \), where \( u(t) \) satisfies \( \Psi(t)u'(t) = g(t) \), or

\[
\begin{pmatrix} e^{-3t} & e^{-t} \\ -e^{-3t} & e^{-t} \end{pmatrix}
\begin{pmatrix} u'_1 \\ u'_2 \end{pmatrix} = \begin{pmatrix} 2e^{-t} \\ 3t \end{pmatrix}.
\] (37)

Solving Eq. (37) by row reduction, we obtain

\[
u'_1 = e^{2t} - \frac{3}{2} e^{3t},
\]

\[
u'_2 = 1 + \frac{3}{2} e^t.
\]

Hence

\[
u_1(t) = \frac{1}{2} e^{2t} - \frac{3}{2} e^{3t} + \frac{1}{6} e^{3t} + c_1,
\]

\[
u_2(t) = t + \frac{3}{2} e^t - \frac{3}{2} e^t + c_2,
\]

and

\[
x = \Psi(t)u(t)
\]

\[
= c_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-3t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} te^{-t} + \frac{1}{2} \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} t - \frac{1}{3} \begin{pmatrix} 4 \\ 5 \end{pmatrix},
\] (38)

which is the same as the solution obtained previously.

Each of the methods for solving nonhomogeneous equations has some advantages and disadvantages. The method of undetermined coefficients requires no integration, but is limited in scope and may entail the solution of several sets of algebraic equations. The method of diagonalization requires finding the inverse of the transformation matrix and the solution of a set of uncoupled first order linear equations, followed by a matrix multiplication. Its main advantage is that for Hermitian coefficient matrices the inverse of the transformation matrix can be written down without calculation, a feature that is more important for large systems. Variation of parameters is the most general method. On the other hand, it involves the solution of a set of linear algebraic equations with variable coefficients, followed by an integration and a matrix multiplication, so it may also be the most complicated from a computational viewpoint. For many small systems