(b) If \( I(t) = e^{-t/2} \), determine the solution of the system (i) that also satisfies the initial conditions \( x(0) = 0 \).

In each of Problems 14 and 15 verify that the given vector is the general solution of the corresponding homogeneous system, and then solve the nonhomogeneous system. Assume that \( t > 0 \).

14. \[ \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} x + \begin{pmatrix} 1 - t^2 \\ 2t \end{pmatrix}, \quad x(0) = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} t + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} t^{-1} \]

15. \[ \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} x + \begin{pmatrix} 5t \\ t^3 - 1 \end{pmatrix}, \quad x(0) = c_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} t^{-1} + c_2 \begin{pmatrix} 2 \\ 5 \end{pmatrix} t^2 \]

16. Let \( x = \Phi(t) \) be the general solution of \( \dot{x} = P(t)x + g(t) \), and let \( x = v(t) \) be some particular solution of the same system. By considering the difference \( \Phi(t) - v(t) \), show that \( \Phi(t) = u(t) + v(t) \), where \( u(t) \) is the general solution of the homogeneous system \( \dot{x} = P(t)x \).

17. Consider the initial value problem

\[ \dot{x} = Ax + g(t), \quad x(0) = x^0. \]

(a) By referring to Problem 15(c) in Section 7.7, show that

\[ x = \Phi(t)x^0 + \int_0^t \Phi(t-s)g(s) \, ds. \]

(b) Show also that

\[ x = \exp(At)x^0 + \int_0^t \exp(At-s)g(s) \, ds. \]

Compare these results with those of Problem 27 in Section 3.7.

REFERENCES

Further information on matrices and linear algebra is available in any introductory book on the subject. The following is a representative sample:


