58 Are there patterns A and B of heads and tails such that A is longer than B, yet A appears before B more than half the time when a fair coin is being flipped?

59 Let k and n be fixed positive integers with k < n.
   a Find a closed form for the probability generating function

   \[ G(w, z) = \frac{1}{m^n} \sum_{h_1 = 1}^{m} \ldots \sum_{h_n = 1}^{m} w^{p(h_1, \ldots, h_n; k)} z^{p(h_1, \ldots, h_n; n)} \]

   for the joint distribution of the numbers of probes needed to find the kth and nth items that have been inserted into a hash table with m lists.
   b Although the random variables \( P(h_1, \ldots, h_n; k) \) and \( P(h_1, \ldots, h_n; n) \) are dependent, show that they are somewhat independent:

   \[ E(P(h_1, \ldots, h_n; k)P(h_1, \ldots, h_n; n)) = \left( E(P(h_1, \ldots, h_n; k)) E(P(h_1, \ldots, h_n; n)) \right) \]

60 Use the result of the previous exercise to prove (8.103).

61 Continuing exercise 47, find the variance of the number of diphages after n irradiations.

Research problems

62 The normal distribution is a non-discrete probability distribution characterized by having all its cumulants zero except the mean and the variance. Is there an easy way to tell if a given sequence of cumulants \((\kappa_1, \kappa_2, \kappa_3, \ldots)\) comes from a discrete distribution? (All the probabilities must be “atomic” in a discrete distribution.)

63 Is there any sequence \( A = \tau_1 \tau_2 \ldots \tau_{l-1} \tau_l \) of \( l \geq 3 \) heads and tails such that the sequences \( H\tau_1 \tau_2 \ldots \tau_{l-1} \) and \( T\tau_1 \tau_2 \ldots \tau_{l-1} \) both perform equally well against A in the game of Penney ante?