if at all. But the right-hand column shows that $P(n)$ is very close indeed to $\sqrt{\pi n}/2$. Thus we can characterize the behavior of $P(n)$ much better if we can derive formulas of the form

$$P(n) = \sqrt{\pi n}/2 + O(1),$$

or even sharper estimates like

$$P(n) = \sqrt{\pi n}/2 - \frac{1}{2} + O(1/\sqrt{n}).$$

Stronger methods of asymptotic analysis are needed to prove $O$-results, but the additional effort required to learn these stronger methods is amply compensated by the improved understanding that comes with $O$-bounds.

Moreover, many sorting algorithms have running times of the form

$$T(n) = An \log n + Bn + O(\log n)$$

for some constants $A$ and $B$. Analyses that stop at $T(n) \sim An \log n$ don’t tell the whole story, and it turns out to be a bad strategy to choose a sorting algorithm based just on its $A$ value. Algorithms with a good ‘$A$’ often achieve this at the expense of a bad ‘$B$’. Since $\log n$ grows only slightly faster than $n$, the algorithm that’s faster asymptotically (the one with a slightly smaller $A$ value) might be faster only for values of $n$ that never actually arise in practice. Thus, asymptotic methods that allow us to go past the first term and evaluate $B$ are necessary if we are to make the right choice of method.

Before we go on to study $O$, let’s talk about one more small aspect of mathematical style. Three different notations for logarithms have been used in this chapter: $\lg$, $\ln$, and $\log$. We often use ‘$\lg$’ in connection with computer methods, because binary logarithms are often relevant in such cases; and we often use ‘$\ln$’ in purely mathematical calculations, since the formulas for natural logarithms are nice and simple. But what about ‘$\log$’? Isn’t this the “common” base-10 logarithm that students learn in high school-the “common” logarithm that turns out to be very uncommon in mathematics and computer science? Yes; and many mathematicians confuse the issue by using ‘$\log$’ to stand for natural logarithms or binary logarithms. There is no universal agreement here. But we can usually breathe a sigh of relief when a logarithm appears inside $O$-notation, because $O$ ignores multiplicative constants. There is no difference between $O(\log n)$, $O(\ln n)$, and $O(\log n)$, as $n \to \infty$; similarly, there is no difference between $0 (\lg \lg n)$, $0 (\ln \ln n)$, and $O(\log \log n)$. We get to choose whichever we please; and the one with ‘$\log$’ seems friendlier because it is more pronounceable. Therefore we generally use ‘$\log$’ in all contexts where it improves readability without introducing ambiguity.