but we determine the coefficients by using the points \((t_n, y_n)\) and \((t_{n+1}, y_{n+1})\). Thus \(\alpha\) and \(\beta\) must satisfy

\[
\alpha t_n + \beta = f_n, \\
\alpha t_{n+1} + \beta = f_{n+1},
\]

and it follows that

\[
\alpha = \frac{f_{n+1} - f_n}{h}, \quad \beta = \frac{f_n t_{n+1} - f_{n+1} t_n}{h}. \tag{8}
\]

Substituting \(Q_2(t)\) for \(\phi'(t)\) in Eq. (2) and simplifying, we obtain

\[
y_{n+1} = y_n + \frac{1}{2} h f_n + \frac{1}{2} h f(t_{n+1}, y_{n+1}), \tag{9}
\]

which is the second order Adams–Moulton formula. We have written \(f(t_{n+1}, y_{n+1})\) in the last term to emphasize that the Adams–Moulton formula is implicit, rather than explicit, since the unknown \(y_{n+1}\) appears on both sides of the equation. The local truncation error for the second order Adams–Moulton formula is proportional to \(h^3\).

The first order Adams–Moulton formula is just the backward Euler formula, as you might anticipate by analogy with the first order Adams–Bashforth formula.

More accurate higher order formulas can be obtained by using an approximating polynomial of higher degree. The fourth order Adams–Moulton formula, with a local truncation error proportional to \(h^5\), is

\[
y_{n+1} = y_n + \frac{h}{24}(9 f_{n+1} + 19 f_n - 5 f_{n-1} + f_{n-2}). \tag{10}
\]

Observe that this is also an implicit formula because \(y_{n+1}\) appears in \(f_{n+1}\).

Although both the Adams–Bashforth and Adams–Moulton formulas of the same order have local truncation errors proportional to the same power of \(h\), the Adams–Moulton formulas of moderate order are in fact considerably more accurate. For example, for the fourth order formulas (6) and (10), the proportionality constant for the Adams–Moulton formula is less than 1/10 of the proportionality constant for the Adams–Bashforth formula. Thus the question arises: Should one use the explicit (and faster) Adams–Bashforth formula, or the more accurate but implicit (and slower) Adams–Moulton formula? The answer depends on whether by using the more accurate formula one can increase the step size, and therefore reduce the number of steps, enough to compensate for the additional computations required at each step.

In fact, numerical analysts have attempted to achieve both simplicity and accuracy by combining the two formulas in what is called a \textbf{predictor–corrector method}. Once \(y_{n-3}, y_{n-2}, y_{n-1}\), and \(y_n\) are known, we can compute \(f_{n-3}, f_{n-2}, f_{n-1}\), and \(f_n\), and then use the Adams–Bashforth (predictor) formula (6) to obtain a first value for \(y_{n+1}\). Then we compute \(f_{n+1}\) and use the Adams–Moulton (corrector) formula (10), which is no longer implicit, to obtain an improved value of \(y_{n+1}\). We can, of course, continue to use the corrector formula (10) if the change in \(y_{n+1}\) is too large. However, if it is necessary to use the corrector formula more than once or perhaps twice, it means that the step size \(h\) is too large and should be reduced.

In order to use any of the multistep methods it is necessary first to calculate a few \(y_j\) by some other method. For example, the fourth order Adams–Moulton method requires values for \(y_1\) and \(y_2\), while the fourth order Adams–Bashforth method also requires a value for \(y_3\). One way to proceed is to use a one-step method of comparable accuracy to calculate the necessary starting values. Thus, for a fourth order multistep method,