Comparing the calculated value with the exact value \( \phi(0.4) = 5.7942260 \), we find that the error is 0.0025366. This is somewhat better than the result using the Adams–Bashforth method, but not as good as the result using the predictor–corrector method, and not nearly as good as the result using the Adams–Moulton method.

A comparison between one-step and multistep methods must take several factors into consideration. The fourth order Runge–Kutta method requires four evaluations of \( f \) at each step, while the fourth order Adams–Bashforth method (once past the starting values) requires only one and the predictor–corrector method only two. Thus, for a given step size \( h \), the latter two methods may well be considerably faster than Runge–Kutta. However, if Runge–Kutta is more accurate and therefore can use fewer steps, then the difference in speed will be reduced and perhaps eliminated. The Adams–Moulton and backward differentiation formulas also require that the difficulty in solving the implicit equation at each step be taken into account. All multistep methods have the possible disadvantage that errors in earlier steps can feed back into later calculations with unfavorable consequences. On the other hand, the underlying polynomial approximations in multistep methods make it easy to approximate the solution at points between the mesh points, should this be desirable. Multistep methods have become popular largely because it is relatively easy to estimate the error at each step and to adjust the order or the step size to control it. For a further discussion of such questions as these see the books listed at the end of the chapter; in particular, Shampine (1994) is an authoritative source.

**PROBLEMS**

In each of Problems 1 through 6 determine an approximate value of the solution at \( t = 0.4 \) and \( t = 0.5 \) using the specified method. For starting values use the values given by the Runge–Kutta method; see Problems 1 through 6 of Section 8.3. Compare the results of the various methods with each other and with the actual solution (if available).

(a) Use the fourth order predictor–corrector method with \( h = 0.1 \). Use the corrector formula once at each step.
(b) Use the fourth order Adams–Moulton method with \( h = 0.1 \).
(c) Use the fourth order backward differentiation method with \( h = 0.1 \).

1. \( y' = 3 + t - y \), \( y(0) = 1 \)
2. \( y' = 5t - 3\sqrt{y} \), \( y(0) = 2 \)
3. \( y' = 2y - 3t \), \( y(0) = 1 \)
4. \( y' = 2t + e^{-ty} \), \( y(0) = 1 \)
5. \( y' = \frac{y^2 + 2ty}{3 + t^2} \), \( y(0) = 0.5 \)
6. \( y' = (t^2 - y^2) \sin y \), \( y(0) = -1 \)

In each of Problems 7 through 12 find approximate values of the solution of the given initial value problem at \( t = 0.5, 1.0, 1.5, \) and 2.0, using the specified method. For starting values use the values given by the Runge–Kutta method; see Problems 7 through 12 in Section 8.3. Compare the results of the various methods with each other and with the actual solution (if available).

(a) Use the fourth order predictor–corrector method with \( h = 0.05 \). Use the corrector formula once at each step.
(b) Use the fourth order Adams–Moulton method with \( h = 0.05 \).
(c) Use the fourth order backward differentiation method with \( h = 0.05 \).

7. \( y' = 0.5 - t + 2y \), \( y(0) = 1 \)
8. \( y' = 5t - 3\sqrt{y} \), \( y(0) = 2 \)
9. \( y' = \sqrt{t + y} \), \( y(0) = 3 \)
10. \( y' = 2t + e^{-ty} \), \( y(0) = 1 \)