9 Prove (9.22) rigorously by showing that the left side is a subset of the right side, according to the set-of-functions definition of 0.

10 Prove or disprove: \( \cos O(x) = 1 + O(x^2) \) for all real \( x \).

11 Prove or disprove: \( O(x + y)^2 = O(x^2) + O(y^2) \).

12 Prove that
\[
1 + \frac{2}{n} + O(n^{-2}) = \left(1 + \frac{2}{n}\right) \left(1 + O(n^{-2})\right),
\]
as \( n \to \infty \).

13 Evaluate \( (n + 2 + O(n^{-1}))^n \) with relative error \( O(n^{-1}) \).

14 Prove that \( (n + \alpha)^{n+\beta} = n^n e^{\beta (1 + \alpha (\beta - \frac{1}{2} \alpha) n^{-1} + O(n^{-2}))} \).

15 Give an asymptotic formula for the “middle” trinomial coefficient \( \binom{n}{r, r, n} \), correct to relative error \( O(n^{-3}) \).

16 Show that if \( B(1 - x) = -B(x) \geq 0 \) for \( 0 < x < \frac{1}{2} \), we have
\[
\int_0^b B([x]) f(x) \, dx = 0
\]
if we assume also that \( f'(x) \geq 0 \) for \( a \leq x \leq b \).

17 Use generating functions to show that \( B_m(\frac{1}{2}) = (2^{1-m} - 1)B_m \), for all \( m \geq 0 \).

18 Find \( \sum_k \left( \frac{2^n}{k} \right)^k \) with relative error \( O(n^{-1/4}) \), when \( a > 0 \).

**Homework exercises**

19 Use a computer to compare the left and right sides of the approximations in Table 438, when \( n = 10 \), \( z = \alpha = 0.1 \), and \( O(f(n)) = O(f(z)) = 0 \).

20 Prove or disprove the following estimates, as \( n \to \infty \):
\[
\begin{align*}
a &\quad O\left(\left(\frac{n^2}{\log \log n}\right)^{1/2}\right) \leq O(\sqrt{n})^2. \\
b &\quad e^{O(1/n)} = e + O(1/n). \\
c &\quad n! = O((1 - 1/n)^n n^n). \\
\end{align*}
\]

21 Equation (9.48) gives the nth prime with relative error \( O(\log n)^{-2} \). Improve the relative error to \( O(\log n)^{-3} \) by starting with another term of (9.31) in (9.46).

22 Improve (9.54) to \( O(n^{-3}) \).

23 Push the approximation (9.62) further, getting absolute error \( O(n^{-3}) \). Hint: Let \( g_n = c/(n + 1) (n + 2) + h_n \), what recurrence does \( h_n \) satisfy?