which states that the time rate of change of angular momentum about any point is equal to the moment of the resultant force about that point. The angular momentum about the origin is $mL^2(d\theta/dt)$, so the governing equation is

$$mL^2 \frac{d^2\theta}{dt^2} = -cL \frac{d\theta}{dt} - mgL \sin\theta. \quad (10)$$

The factors $L$ and $L \sin\theta$ on the right side of Eq. (10) are the moment arms of the resistive force and of the gravitational force, respectively, while the minus signs are due to the fact that the two forces tend to make the pendulum rotate in the clockwise (negative) direction. You should verify, as an exercise, that the same equation is obtained for the other three possible sign combinations of $\theta$ and $d\theta/dt$.

By straightforward algebraic operations we can write Eq. (10) in the standard form

$$\frac{d^2\theta}{dt^2} + \frac{c}{mL} \frac{d\theta}{dt} + \frac{g}{L} \sin\theta = 0, \quad (11)$$

or

$$\frac{d^2\theta}{dt^2} + \gamma \frac{d\theta}{dt} + \omega^2 \sin\theta = 0, \quad (12)$$

where $\gamma = c/mL$ and $\omega^2 = g/L$. To convert Eq. (12) to a system of two first order equations we let $x = \theta$ and $y = d\theta/dt$; then

$$\frac{dx}{dt} = y, \quad \frac{dy}{dt} = -\omega^2 \sin x - \gamma y. \quad (13)$$

Since $\gamma$ and $\omega^2$ are constants, the system (13) is an autonomous system of the form (1).

The critical points of Eqs. (13) are found by solving the equations

$$y = 0, \quad -\omega^2 \sin x - \gamma y = 0.$$

We obtain $y = 0$ and $x = \pm n\pi$, where $n$ is an integer. These points correspond to two physical equilibrium positions, one with the mass directly below the point of support ($\theta = 0$) and the other with the mass directly above the point of support ($\theta = \pi$). Our intuition suggests that the first is stable and the second is unstable.