Find a “combinatorial” proof of Stirling’s approximation. (Note that $n^n$ is the number of mappings of $\{1, 2, \ldots, n\}$ into itself, and $n!$ is the number of mappings of $\{1, 2, \ldots, n\}$ onto itself.)

Consider an $n \times n$ array of dots, $n \geq 3$, in which each dot has four neighbors. (At the edges we “wrap around” modulo $n$.) Let $X_n$ be the number of ways to assign the colors red, white, and blue to these dots in such a way that no neighboring dots have the same color. (Thus $X_3 = 12$.)

Prove that

$$X_n \sim \left(\frac{4}{3}\right)^{3n/2} e^{-\pi/6}.$$ 

Let $Q_n$ be the least integer $m$ such that $H_m > n$. Find the smallest integer $n$ such that $Q_n \neq \left[e^{n-\gamma} + \frac{1}{2}\right]$, or prove that no such $n$ exist.

Th-th-th-that’s all, folks!