Research problems

65 Find a “combinatorial” proof of Stirling’s approximation. (Note that \( n^n \) is the number of mappings of \( \{1, 2, \ldots, n\} \) into itself, and \( n! \) is the number of mappings of \( \{1, 2, \ldots, n\} \) onto itself.)

66 Consider an \( n \times n \) array of dots, \( n \geq 3 \), in which each dot has four neighbors. (At the edges we “wrap around” modulo \( n \).) Let \( \chi_n \) be the number of ways to assign the colors red, white, and blue to these dots in such a way that no neighboring dots have the same color. (Thus \( \chi_3 = 12 \).) Prove that

\[
\chi_n \sim \left( \frac{4}{3} \right)^{3n/2} e^{-\pi/6}.
\]

67 Let \( Q_n \) be the least integer \( m \) such that \( H_m > n \). Find the smallest integer \( n \) such that \( Q_n \neq \left\lfloor e^{n-\gamma} + \frac{1}{2} \right\rfloor \), or prove that no such \( n \) exist.

Th-th-th-that’s all, folks!