Venn \[294\] claimed that there is no way to do the five-set case with ellipses, but a five-set construction with ellipses was found by Grünbaum \[137\].

1.6 If the \( n \)th line intersects the previous lines in \( k > 0 \) distinct points, we get \( k-1 \) new bounded regions (assuming that none of the previous lines were mutually parallel) and two new infinite regions. Hence the maximum number of bounded regions is \((n-2)+(n-3)+\cdots = S_{n-2} = (n-1)(n-2)/2 = L_n - 2n\).

1.7 The basis is unproved; and in fact, \( H(1) \neq 2 \).

1.8 \( Q_2 = (1 + \beta)/\alpha; \) \( Q_3 = (1 + \alpha + \beta)/\alpha\beta; \) \( Q_4 = (1 + \alpha)/\beta; \) \( Q_5 = \alpha; \) \( Q_6 = \beta \). So the sequence is periodic!

1.9 (a) We get \( P(n-1) \) from the inequality

\[
\frac{x_1 \ldots x_{n-1}}{n-1} \leq \left( \frac{x_1 + \cdots + x_{n-1}}{n-1} \right)^n.
\]

(b) \( x_1 \ldots x_n x_{n+1} \ldots x_{2n} \leq \left( \frac{(x_1 + \cdots + x_n)(x_{n+1} + \cdots + x_{2n})}{n} \right)^n \) by \( P(n); \) the product inside is \( \leq \left( \frac{x_1 + \cdots + x_{2n}}{2n} \right)^n \) by \( P(2) \). (c) For example, \( P(5) \) follows from \( P(6) \) from \( P(3) \) from \( P(4) \) from \( P(2) \).

1.10 First show that \( R_n = R_{n-1} + 1 + Q_{n-1} + 1 + R_{n-1} \), when \( n > 0 \). Incidentally, the methods of Chapter 7 will tell us that \( Q_n = ((1 + \sqrt{3})^{n+1} - (1 - \sqrt{3})^{n+1})/(2\sqrt{3}) - 1 \).

1.11 (a) We cannot do better than to move a double \((n-1)\)-tower, then move (and invert the order of) the two largest disks, then move the double \((n-1)\)-tower again; hence \( A_n = 2A_{n-1} + 2 \) and \( A_n = 2T_n = 2^{n+1} - 2 \). This solution interchanges the two largest disks but returns the other \( 2n - 2 \) to their original order.

(b) Let \( B_n \) be the minimum number of moves. Then \( B_1 = 3 \), and it can be shown that no strategy does better than \( B_n = A_{n-1} + 2 + A_{n-1} + 2 + B_{n-1} \), when \( n \geq 1 \). Hence \( B_n = 2^{n+2} - 5 \), for all \( n \geq 0 \). Curiously this is just \( 2A_{n-1} \), and we also have \( B_n = A_{n-1} + 1 + A_{n-1} + 1 + A_{n-1} + 1 + A_{n-1} + 1 \).

1.12 If all \( m_k > 0 \), then \( A(m_1, \ldots, m_n) = 2A(m_1, \ldots, m_{n-1}) + m_n \). This is an equation of the “generalized Josephus” type, with solution \((m_1, \ldots, m_n) = 2^{n-1}m_1 + \cdots + 2m_{n-1} + m_n \).

Incidentally, the corresponding generalization of exercise \( \text{I.1b} \) appears to satisfy the recurrence

\[
B(m_1, \ldots, m_n) = \begin{cases} 
A(m_1, \ldots, m_n), & \text{if } m_n = 1; \\
2m_n - 1, & \text{if } n = 1; \\
2A(m_1, \ldots, m_{n-1}) + 2m_n + B(m_1, \ldots, m_{n-1}), & \text{if } n > 1 \text{ and } m_n > 1.
\end{cases}
\]

This answer assumes that \( n > 0 \).