2.24 Summing by parts, \( \sum x^m H_x \delta x = x^{m+1} H_x / (m+1) - x^{m+1} / (m+1)^2 + C \); hence \( \sum_{0 \leq k < n} k^m H_k = n^{m+1} / (m+1) - x^{m+1} / (m+1)^2 + C \). In our case \( m = -2 \), so the sum comes to \( 1 - (n+1) / (n+1) \).

2.25 Here are some of the basic analogies:

\[
\sum_{k \in K} c_{a_k} = c \sum_{k \in K} a_k \quad \iff \quad \prod_{k \in K} a_k^c = \left( \prod_{k \in K} a_k \right)^c
\]

\[
\sum_{k \in K} (a_k + b_k) = \sum_{k \in K} a_k + \sum_{k \in K} b_k \quad \iff \quad \prod_{k \in K} a_k \times \prod_{k \in K} b_k = \left( \prod_{k \in K} a_k \right) \left( \prod_{k \in K} b_k \right)
\]

\[
\sum_{k \in K} a_k = \sum_{p \mid k \in K} a_\{p \mid k\} \quad \iff \quad \prod_{k \in K} a_k = \prod_{p \mid k \in K} a_{p \mid k}
\]

\[
\sum_{i \in I \mid k \in K} a_{i \mid k} = \sum_{i \in I \mid k \in K} a_{i \mid k} \quad \iff \quad \prod_{i \in I \mid k \in K} a_{i \mid k} = \prod_{i \in I \mid k \in K} a_{i \mid k}
\]

\[
\sum_{k \in K} \frac{1}{k} = \#K \quad \iff \quad \prod_{k \in K} c = c \#K
\]

2.26 \( P^2 = \left( \prod_{1 \leq j, k \leq n} a_j a_k \right) \left( \prod_{1 \leq j = k \leq n} a_j a_k \right) \). The first factor is \( \left( \prod_{k = 1}^{n} a_k^2 \right)^2 \); the second factor is \( \prod_{k = 1}^{n} a_k^2 \). Hence \( P = \left( \prod_{k = 1}^{n} a_k \right)^{n+1} \).

2.27 \( \Delta(c^x) = c^x (c - x - 1) = c^{x+2} / (c - x) \). Setting \( c = -2 \) and decreasing \( x \) by \( 2 \) yields \( \Delta(-(-x-2)) = (-2)^x / x \), hence the stated sum is \( (-2)^{-2} = (-2)^{-1} = (-1)^n n! = 1 \).

2.28 The interchange of summation between the second and third lines is not justifiable; the terms of this sum do not converge absolutely. Everything else is perfectly correct, except that the result of \( \sum_{k \geq 1} |k = j - 1| k/j \) should perhaps have been written \( [j - 1 \geq 1] / (j - 1)/j \) and simplified explicitly. As opposed to imperfectly correct.

2.29 Use partial fractions to get

\[
\frac{k}{4k^2 - 1} = \frac{1}{4} \left( \frac{1}{2k + 1} + \frac{1}{2k - 1} \right)
\]

The \( (-1)^k \) factor now makes the two halves of each term cancel with their neighbors. Hence the answer is \(-1/4 + (-1)^n / (8n + 4)\).

2.30 \( \sum_{a} a \cdot x \cdot dx = \frac{1}{2} (b^2 - a^2) = \frac{1}{2} (b - a) (b + a - 1) \). Hence we have

\[
(b - a) (b + a - 1) = 2100 \quad = 2^2 \cdot 3 \cdot 5^2 \cdot 7.
\]