so the slope of the graph of Fruit against Root for the ungrazed plants is given by

\[ b_u = \frac{SSXY_u}{SSX_u} = \frac{350.0302}{14.58677} = 23.996 \]

Now add up the regression statistics across the factor levels (grazed and ungrazed):

\[ SSY_{g+u} = 11837.79 + 8995.606 = 20833.4, \]
\[ SSX_{g+u} = 19.9111 + 14.58677 = 34.49788, \]
\[ SSXY_{g+u} = 462.7415 + 350.0302 = 812.7717, \]
\[ SSR_{g+u} = 10754.29 + 8399.436 = 19153.75, \]
\[ SSE_{g+u} = 1083.509 + 596.1403 = 1684.641. \]

The SSR for a model with a single common slope is given by

\[ SSR_c = \frac{(SSXY_{g+u})^2}{SSX_{g+u}} = \frac{812.7717^2}{34.49788} = 19148.94, \]

and the value of the single common slope is

\[ b = \frac{SSXY_{g+u}}{SSX_{g+u}} = \frac{812.7717}{34.49788} = 23.560 \]

The difference between the two estimates of SSR (\( SSR_{\text{diff}} = SSR_{g+u} - SSR_c = 19153.75 - 19148.94 = 4.81 \)) is a measure of the significance of the difference between the two slopes estimated separately for each factor level. Finally, SSE is calculated by difference:

\[ SSE = SSY - SSA - SSR_c - SSR_{\text{diff}} \]
\[ = 23743.84 - 2910.44 - 19148.94 - 4.81 = 1679.65. \]

Now we can complete the ANOVA table for the full model:

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>d.f.</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grazing</td>
<td>2910.44</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Root</td>
<td>19148.94</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Different slopes</td>
<td>4.81</td>
<td>1</td>
<td>4.81</td>
<td>n.s.</td>
</tr>
<tr>
<td>Error</td>
<td>1679.65</td>
<td>36</td>
<td>46.66</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>23743.84</td>
<td>39</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Degrees of freedom for error are 40 – 4 = 36 because we have estimated four parameters from the data: two slopes and two intercepts. So the error variance is 46.66 (= SSE/36). The difference between the slopes is clearly not significant (\( F = 4.81/46.66 = 0.10 \)) so we
can fit a simpler model with a common slope of 23.56. The sum of squares for differences between the slopes (4.81) now becomes part of the error sum of squares:

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>d.f.</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grazing</td>
<td>2910.44</td>
<td>1</td>
<td>2910.44</td>
<td>63.9291</td>
</tr>
<tr>
<td>Root</td>
<td>19148.94</td>
<td>1</td>
<td>19148.94</td>
<td>420.6156</td>
</tr>
<tr>
<td>Error</td>
<td>1684.46</td>
<td>37</td>
<td>45.526</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>23743.84</td>
<td>39</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This is the minimal adequate model. Both of the terms are highly significant and there are no redundant factor levels.

The next step is to calculate the intercepts for the two parallel regression lines. This is done exactly as before, by rearranging the equation of the straight line to obtain \( a = y - bx \). For each line we can use the mean values of \( x \) and \( y \), with the common slope in each case. Thus:

\[
a_1 = \bar{Y}_1 - b\bar{X}_1 = 50.88 - 23.56 \times 6.0529 = -91.7261, \\
a_2 = \bar{Y}_2 - b\bar{X}_2 = 67.94 - 23.56 \times 8.309 = -127.8294.
\]

This demonstrates that the grazed plants produce, on average, 36.1 mg of fruit less than the ungrazed plants \((127.83 - 91.73)\).

Finally, we need to calculate the standard errors for the common regression slope and for the difference in mean fecundity between the treatments, based on the error variance in the minimal adequate model, given in the table above:

\[
s^2 = \frac{1684.46}{37} = 45.526
\]

The standard errors are obtained as follows. The standard error of the common slope is found in the usual way:

\[
se_b = \sqrt{\frac{s^2}{SSX_{g+u}}} = \sqrt{\frac{45.526}{19.9111 + 14.45667}} = 1.149.
\]

The standard error of the intercept of the regression for the grazed treatment is also found in the usual way:

\[
se_a = \sqrt{s^2 \left[ \frac{1}{n} + \frac{(0 - \bar{x})^2}{SSX_{g+u}} \right]} = \sqrt{45.526 \left[ \frac{1}{20} + \frac{8.3094^2}{34.498} \right]} = 9.664.
\]

It is clear that the intercept of \(-127.829\) is very significantly less than zero \((t = 127.829/9.664 = 13.2)\), suggesting that there is a threshold rootstock size before reproduction can begin. Finally, the standard error of the difference between the elevations of the two lines (the grazing effect) is given by

\[
se_{\hat{y}_u - \hat{y}_g} = \sqrt{s^2 \left[ \frac{2}{n} + \frac{(\bar{X}_1 - \bar{X}_2)^2}{SSX_{g+u}} \right]}
\]