2.24 Summing by parts, \( \sum_{0 \leq k < n} k^m H_k \delta(x) = x^{m+1}/(m+1) - x^{m+1}/(m+1)^2 + C; \) hence \( \sum_{0 \leq k < n} k^m H_k = n^{m+1}((H_n - 1)/(m+1)) + 0^{m+1}/(m+1)^2 \). In our case \( m = -2 \), so the sum comes to \( 1 - (n+1)/(n+1) \).

2.25 Here are some of the basic analogies:

\[
\begin{align*}
\sum_{k \in K} c a_k &= c \sum_{k \in K} a_k & \prod_{k \in K} a_k &= \left( \prod_{k \in K} a_k \right)^c \\
\sum_{k \in K} (a_k + b_k) &= \sum_{k \in K} a_k + \sum_{k \in K} b_k & \prod_{k \in K} a_k b_k &= \left( \prod_{k \in K} a_k \right) \left( \prod_{k \in K} b_k \right) \\
\sum_{k \in K} a_k &= \sum_{p \mid k \in K} a_p | k | & \prod_{k \in K} a_k &= \prod_{p \mid k \in K} a_p | k | \\
\sum_{i \in I} a_{i,k} &= \sum_{i \in I} \sum_{k \in K} a_{i,k} & \prod_{i \in I} \prod_{k \in K} a_{i,k} &= \prod_{i \in I} \prod_{k \in K} a_{i,k} \\
\sum_{k \in K} a_k &= \sum_{k \in K} a_k | k \in K | & \prod_{k \in K} a_k &= \prod_{k \in K} a_k | k \in K | \\
\sum_{k \in K} 1 &= \# K & \prod_{k \in K} c &= c^{\# K} \\
\end{align*}
\]

2.26 \( P^2 = (\prod_{1 \leq i, j \leq n} a_i a_j)(\prod_{1 \leq j = k \leq n} a_j a_k) \). The first factor is \( (\prod_{k=1}^n a_k)^c \); the second factor is \( \prod_{k=1}^n a_k^2 \). Hence \( P = (\prod_{k=1}^n a_k)^{n+1} \).

2.27 \( \Delta(c^n) = c^n | (c - x - 1) = c^{x+2}/(c - x) \). Setting \( c = -2 \) and decreasing \( x \) by \( 2 \) yields \( \Delta(-(-2)^{n-1}) = (-2)^{x}/x \), hence the stated sum is \( (-2)^{n-1} = (-2)^{n-1} = (-1)^n ! n! - 1 \).

2.28 The interchange of summation between the second and third lines is not justifiable; the terms of this sum do not converge absolutely. Everything else is perfectly correct, except that the result of \( \sum_{k \geq 1} | k = j - 1 | k/j \) should perhaps have been written \( [j - 1 \geq 1] | (j - 1) / j \) and simplified explicitly.

2.29 Use partial fractions to get

\[
\frac{k}{4k^2 - 1} = \frac{1}{4} \left( \frac{1}{2k + 1} + \frac{1}{2k - 1} \right)
\]

The \( (-1)^k \) factor now makes the two halves of each term cancel with their neighbors. Hence the answer is \(-1/4 + (-1)^n/(8n + 4)\).

2.30 \( \sum_a^b x \ dx = \frac{1}{2} (b^2 - a^2) = \frac{1}{2} (b - a)(b + a - 1) \). So we have

\[
(b - a)(b + a - 1) = 2 \times 0 = 2^2 \times 3^2 \times 5^2 \times 7.
\]