2.36 (a) By definition, 

\( g(n) - g(n-1) = f(n) \).

(b) By part (a), 

\( g(g(n)) - g(g(n-1)) = nf(n) \).

(c) By part (a) again, 

\[
\sum_{k} f(k) [g(g(n-1)) < k \leq g(g(n))] = n \sum_{j} [g(n-1) < j \leq g(n)].
\]

Colin Mallows observes that the sequence can also be defined by the recurrence 

\[ f(1) = 1; \quad f(n+1) = 1 + f(n+1 - f(f(n))) \]

for \( n \geq 0 \).

2.37 (RLG thinks they probably won’t fit; DEK thinks they probably will; OP is not committing himself.)

3.1 \( m = \lfloor \log n \rfloor; \quad l = n - 2^m = n - 2^{\lfloor \log n \rfloor} \).

3.2 (a) \( [x + .5] \), (b) \( [x - .5] \).

3.3 This is \( \lfloor mn \rfloor \lfloor m\alpha \rfloor n/\alpha = mn - 1 \), since \( 0 < \{m\alpha\} < 1 \).

3.4 Something where no proof is required, only a lucky guess (I guess)

3.5 We have \( \lfloor nx \rfloor = n[x] \iff n[x] \leq \lfloor nx \rfloor < n \lfloor x \rfloor + 1 \iff n \lfloor x \rfloor \leq nx < n \lfloor x \rfloor + 1 \iff \lfloor m/\alpha \rfloor n/\alpha + 1 \), by (3.5(a)), (3.7(a)), (3.7(d)), and (3.8); and this is equivalent to \( n[x] < 1 \), when \( n \) is a positive integer. (Notice that \( n[x] \leq \lfloor nx \rfloor \) for all \( x \) in this case.)

3.6 \( f([x]) = f([x]) \).

3.7 \( \lfloor n/m \rfloor + n \mod m \).

3.8 If all boxes contain \( < \lfloor n/m \rfloor \) objects, then \( n < \lfloor n/m \rfloor - 1 \) \( m \), so \( n/m + 1 \leq \lfloor n/m \rfloor \), contradicting (3.5). The other proof is similar.

3.9 We have \( m/n - l/q = (n \text{ mumble } m)/qn \). The process must terminate, because \( 0 \leq n \text{ mumble } m < m \). The denominators of the representation are strictly increasing, hence distinct, because \( qn/(n \text{ mumble } m) > q \).

3.10 \( \lfloor x + 1/2 \rfloor - \lfloor 2x + 1/4 \rfloor \) is not an integer is the nearest integer to \( x \), if \( \{x\} \neq 1/2 \); otherwise it’s the nearest even integer. (See exercise 2.) Thus the formula gives an “unbiased” way to round.