3.11 If \( n \) is an integer, \( \alpha < n < \beta \iff [\alpha] < n < [\beta] \). The number of integers satisfying \( a < n < b \) when \( a \) and \( b \) are integers is \((b - a - 1)(b > a)\).
We would therefore get the wrong answer if \( \alpha = \beta = \text{integer} \).

3.12 Subtract \([n/m]\) from both sides, by (3.6), getting \([n \mod m]/m] = \([n \mod m + m - 1]/m]\). Both sides are now equal to \([n \mod m > 0]\), since \(0 \leq n \mod m < m\).
A shorter but less direct proof simply observes that the first term in (3.24) must equal the last term in (3.25).

3.13 If they form a partition, the text’s formula for \( N(\alpha, n) \) implies that \( 1/\alpha + 1/\beta = 1 \), because the coefficients of \( n \) in the equation \( N(\alpha, n) + N(\beta, n) = n \) must agree if the equation is to hold for large \( n \). Hence \( \alpha \) and \( \beta \) are both rational or both irrational. If both are irrational, we do get a partition, as shown in the text. If both can be written with numerator \( m \), the value \( m-1 \) occurs in neither spectrum. (However, Golomb [121] has observed that the sets \{\( [n\alpha] \mid n \geq 1 \}\} and \{\([n\beta] - 1 \mid n \geq 1 \}\} always do form a partition, when \( 1/\alpha + 1/\beta = 1 \).)

3.14 It’s obvious if \( ny = 0 \), otherwise true by (3.21) and (3.6).

3.15 Plug in \([mx]\) for \( n \) in (3.24): \([mx] = [x] + [x - \frac{1}{m}] + \cdots + [x - \frac{m-1}{m}].\)

3.16 The formula \( n \mod 3 = 1 + \frac{1}{2}(\omega - 1)\omega^n - (\omega + 2)\omega^{2n} \) can be verified by checking it when \( 0 \leq n < 3 \).
A general formula for \( n \mod m \), when \( m \) is any positive integer, appears in exercise 7.25.

3.17 \[ \sum_{k,l:0 \leq k < m}[1 \leq j \leq x + k/m] = \sum_{k,l:0 \leq k < m}[1 \leq j \leq \lfloor x \rfloor] x \lfloor k \lfloor j - x \rfloor \rfloor = \sum_{l=\lfloor x \rfloor} \sum_{k:0 \leq k \leq m(j - x)} \lfloor m[j] \rfloor = [m(\lfloor x \rfloor - x)] = -[mx] = [mx] - m[\lfloor x \rfloor].\]

3.18 We have
\[
S = \sum_{0 \leq \ell < [n\alpha]} \sum_{k \geq n} [j\alpha^{-1} \leq k < (j + \nu)\alpha^{-1}].
\]
If \( j \leq [n\alpha] - n \leq \nu \), there is no contribution, because \((j + \nu)\alpha^{-1} \leq n\).
Hence \( j = [n\alpha] \) is the only case that matters, and the value in that case equals \([([n\alpha] + \nu)\alpha^{-1}] - n \leq [n\alpha^{-1}].\)

3.19 If and only if \( b \) is an integer. (If \( b \) is an integer, \( \log x \) is a continuous, increasing function that takes integer values only at integer points. If \( b \) is not an integer, the condition fails when \( x = b \).)

3.20 We have \( \sum_{k} kx[\alpha \leq kx \leq \beta] = x \sum_{k} kl[\alpha/k] \leq k \leq [\beta/k]], which sums to \( \frac{1}{2}x([\beta/x]([\beta/x + 1] - ([\alpha/x]([\alpha/x - 1])).\)