**9.4 Competing Species**

![Diagram of critical points and direction field for the system (3).](image)

**FIGURE 9.4.1** Critical points and direction field for the system (3).

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**9.4.1 Competing Species**

$x = 0, y = 0$. This critical point corresponds to a state in which both species die as a result of their competition. By rewriting the system (3) in the form

$$
\frac{dx}{dt} = \begin{pmatrix} 1 & 0 \\ 0 & 0.75 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} x^2 + xy \\ 0.5xy + y^2 \end{pmatrix},
$$

or by setting $X = Y = 0$ in Eq. (7), we see that near the origin the corresponding linear system is

$$
\frac{dx}{dt} = \begin{pmatrix} 1 & 0 \\ 0 & 0.75 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}.
$$

The eigenvalues and eigenvectors of the system (9) are

$$
r_1 = 1, \quad \xi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad r_2 = 0.75, \quad \xi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix},
$$

so the general solution of the system is

$$
\begin{pmatrix} x \\ y \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t + c_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{0.75t}.
$$

Thus the origin is an unstable node of both the linear system (9) and the nonlinear system (8) or (3). In the neighborhood of the origin all trajectories are tangent to the $y$-axis except for one trajectory that lies along the $x$-axis.

$x = 1, y = 0$. This corresponds to a state in which species $x$ survives the competition, but species $y$ does not. The corresponding linear system is

$$
\frac{du}{dt} = \begin{pmatrix} -1 & -1 \\ 0 & 0.25 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}.
$$

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