3.36 The sum is

$$\sum_{k,l,m} 2^{-m} [4^m[m = [\log k] [1 \leq k < 2^m]]$$

$$= \sum_{k,l,m} 2^{-m} [4^m[2^m \leq 1 < 2^{m+1}]][2^l \leq k < 2^{l+1}][0 \leq m \leq n]$$

$$= \sum_{l,m} 4^{-m} [2^m \leq 1 < 2^{m+1}][0 \leq m \leq n]$$

$$= \sum_{m} 2^{-m} [0 \leq m \leq n] = 2(1 - 2^{-n}).$$

3.37 First consider the case $m < n$, which breaks into subcases based on whether $m < \frac{1}{2}n$; then show that both sides change in the same way when $m$ is increased by $n$.

3.38 At most one $x_k$ can be noninteger. Discard all integer $x_k$, and suppose that $n$ are left. When $\{x\} \neq 0$, the average of $\{mx\}$ as $m \to \infty$ lies between $\frac{1}{4}$ and $\frac{1}{2}$; hence $\{mx_1\} + \ldots + \{mx_{n}\} = \{mx_1\} + \ldots + mx_{n}$ cannot have average value zero when $n > 1$.

But the argument just given relies on a difficult theorem about uniform distribution. An elementary proof is possible, sketched here for $n = 2$: Let $P_m$ be the point $\{(mx,my)\}$. Divide the unit square $0 \leq x, y < 1$ into triangular regions $A$ and $B$ according as $x + y < 1$ or $x + y \geq 1$. We want to show that $P_m \in B$ for some $m$, if $\{x\}$ and $\{y\}$ are nonzero. If $P_1 \in B$, we're done. Otherwise there is a disk $D$ of radius $\epsilon > 0$ centered at $P_1$ such that $D \subseteq A$. By Dirichlet's box principle, the sequence $P_1, \ldots, P_N$ must contain two points with $|P_k - P_j| < \epsilon$ and $k > j$, if $N$ is large enough.

It follows that $P_{k-1}$ is within $\epsilon$ of $(1,1) = P_1$; hence $P_{k-1} \in B$.

3.39 Replace $j$ by $b-j$ and add the term $j = 0$ to the sum, so that exercise 15 can be used for the sum on $j$. The result,

$$[x/b^k] \sum [x/b^{k+1}]+b = 1,$$

telescopes when summed on $k$. 

This is really only a level 4 problem, in spite of the way it's stated.