again at the general system (2). There are four cases to be considered, depending on the relative orientation of the lines

\[ \epsilon_1 - \sigma_1 x - \alpha_1 y = 0 \quad \text{and} \quad \epsilon_2 - \sigma_2 y - \alpha_2 x = 0, \]

as shown in Figure 9.4.5. These lines are called the \textbf{x} and \textbf{y} \textbf{nullclines}, respectively, because \( x' \) is zero on the first and \( y' \) is zero on the second. Let \((X, Y)\) denote any critical point in any one of the four cases. As in Examples 1 and 2 the system (2) is almost linear in the neighborhood of this point because the right side of each differential equation is a quadratic polynomial. To study the system (2) in the neighborhood of this critical point we can look at the corresponding linear system obtained from Eq. (13) of Section 9.3,

\[
\frac{d}{dt} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \epsilon_1 - 2\sigma_1 X - \alpha_1 Y & -\alpha_1 X \\ -\alpha_2 Y & \epsilon_2 - 2\sigma_2 Y - \alpha_2 X \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}.
\]

We now use Eq. (35) to determine the conditions under which the model described by Eqs. (2) permits the coexistence of the two species \( x \) and \( y \). Of the four possible cases shown in Figure 9.4.5 coexistence is possible only in cases (c) and (d). In these cases the nonzero values of \( X \) and \( Y \) are readily obtained by solving the algebraic equations (34); the result is

\[
X = \frac{\epsilon_1 \sigma_2 - \epsilon_2 \alpha_1}{\sigma_1 \sigma_2 - \alpha_1 \alpha_2}, \quad Y = \frac{\epsilon_2 \sigma_1 - \epsilon_1 \alpha_2}{\sigma_1 \sigma_2 - \alpha_1 \alpha_2}.
\]

**FIGURE 9.4.5** The various cases for the competing species system (2).