5.16 This is just \((2a)! \cdot (2b)! \cdot (2c)! / ((a + b)! \cdot (b + c)! \cdot (c + a)! times \((5.2g)\)\), if we write the summands in terms of factorials.

5.17 \(\binom{2n-1}{2n} = \frac{(4n)/2^{2n}}{(2n-1)/2^{2n}} = \frac{(4n)/2^{2n}}{2^{2n}}\).

5.18 \((\frac{3r}{k})^n / 3^k\).

5.19 \(B_{2n+1}(z) = \sum_{k=0}^{\infty} \binom{k}{k} (-1)^k \frac{1}{(k+1)} \right) (-z)^k\), by \((5.60)\), and this is \(\sum_{k=0}^{\infty} \binom{k}{k} \frac{1}{(k+1)} \right) (-z)^k = B_t(z)\).

5.20 It equals \(F(-a_1, \ldots, -a_n; -b_1, \ldots, -b_m; -1)^m+nz\); see exercise 2.17.

5.21 \(\lim_{n \to \infty} (n + m) / n^m = 1\).

5.22 Multiplying and dividing instances of \((5.83)\) gives

\[
\frac{(-1/2)!}{x! (x - 1/2)!} = \lim_{n \to \infty} \left( \frac{n+x}{n} \right) \left( \frac{n+x-1/2}{n} \right) n^{2x} \left/ \left( \frac{n-1/2}{n} \right) \right.
\]

\[
= \lim_{n \to \infty} \left( 2n + 2x \right) \left/ \left( 2n \right) \right. n^{-2x},
\]

by \((5.34)\) and \((5.36)\). Also

\[
\frac{1}{2x!} = \lim_{n \to \infty} \left( \frac{2n + 2x}{2n} \right) \left/ \left( 2n \right) \right. 2^{2x}.1
\]

Hence, etc. The Gamma function equivalent, incidentally, is

\[
\Gamma(x) \Gamma(x + \frac{1}{2}) = \Gamma(2x) \Gamma(\frac{1}{2}) / 2^{2x}.1
\]

5.23 \((-1)^n \cdot n!\), see \((5.50)\).

5.24 This sum is \(\binom{n}{m} F_{1/2}^{m-n-m+1} \right) = \binom{2n}{m}\), by \((5.35)\) and \((5.93)\).

5.25 This is equivalent to the easily proved identity

\[
(a - b) \frac{a^k}{(b + 1)^k} = \frac{(a+1)^k}{(b+1)^k} - b \frac{a^k}{b^k}
\]

as well as to the operator formula \(a \cdot b = (4 + a) \cdot (4 + b)\).

Similarly, we have

\[
(a_1 \cdot a_2) F\left( \frac{a_1, a_2, a_3, \ldots, a_m}{b_1, \ldots, b_n} \left| z \right. \right)
\]

\[
= a_1 F\left( \frac{a_1+1, a_2, a_3, \ldots, a_m}{b_1, \ldots, b_n} \left| z \right. \right) - a_2 F\left( \frac{a_1, a_2+1, a_3, \ldots, a_m}{b_1, \ldots, b_n} \left| z \right. \right).
\]