If the harmonic numbers are worm numbers, the Fibonacci numbers are rabbit numbers.

**A Answers to Exercises 527**

**6.7** (a) Set \( k = 1 - n \) and apply (6.107). (b) Set \( m = 1 \) and \( k = n - 1 \) and apply (6.128).

**6.8** \( 55 + 8 + 2 \) becomes \( 89 + 13 + 3 = 105 \); the true value is \( 104,607361 \).

**6.9** \( 21 \). (We go from \( F_n \) to \( F_{n+2} \) when the units are squared. The true answer is about 20.72.)

**6.10** The partial quotients \( a_0, a_1, a_2, \ldots \) are all equal to 1, because \( \phi = 1 + 1 / \phi \). (The Stern-Brocot representation is therefore RLRLRLRLRL... . )

**6.11** \((-1)^n = [n = 0] - [n = 1] \); see (6.11).

**6.12** This is a consequence of (6.31) and its dual in Table 250.

**6.13** The two formulas are equivalent, by exercise 12. We can use induction. Or we can observe that \( z^n D^n \) applied to \( f(z) = z^k \) gives \( x^n z^k \); while \( D^n \) applied to the same function gives \( x^n z^k \); therefore the sequence \( \langle 0, 0, 0, \ldots \rangle \) must relate to \( \langle 0^n, 0^1, 0^2, \ldots \rangle \) as \( \langle x^0, x^1, x^2, \ldots \rangle \) relates to \( \langle x^0, x^1, x^2, \ldots \rangle \).

**6.14** We have

\[
\binom{x + k}{n} = \frac{(k + 1) \binom{x + k}{n + 1} + (n - k) \binom{x + k + 1}{n + 1}}{n + 1},
\]

because \( (n+1)x = (k+1)(x+k-n)+(n-k)(x+k+1) \). (It suffices to verify the latter identity when \( k = 0, k = -1 \), and \( k = n \).)

**6.15** Since \( \Delta\binom{x+k}{n} = \binom{x+k}{n-1} \), we have the general formula

\[
\sum_k \binom{n}{k} \binom{x + k}{n - m} = \Delta^m \left[ x^n \right] = \sum_j (-1)^{m-j} (x + j)^m.
\]

Set \( x = 0 \) and appeal to (6.19).

**6.16** \( A_{n,k} = \sum_{j \geq 0} a_{j \{ \frac{n-j}{k} \}} \); this sum is always finite.

**6.17** (a) \( \binom{n}{k} \) \( \binom{n-1}{k} \). (b) \( \binom{n}{k} \) = \( n \binom{n-1}{k} \) \( n! \geq k! / k! \). (c) \( \binom{n}{k} = k! \binom{n}{k} \).

**6.18** This is equivalent to (6.3) or (6.8). (It follows in particular that \( a_0(t) = -n! \sigma_n(0) = B_n / n! \) when \( n > 1 \).)

**6.19** Use Table 258.

**6.20** \( \sum_{1 \leq j \leq n} \frac{1}{j^2} = \sum_{1 \leq j \leq n} (n + 1 - j) / j^2 = (n + 1) \zeta(2) \).