6.74 If $p(x)$ is any polynomial of degree $\leq n$, we have
\[
p(x) = \sum_k p(-k) \binom{-x}{k} \binom{x + n}{n - k},
\]
because this equation holds for $x = 0, -1, \ldots, -n$. The stated identity is the special case where $p(x) = \chi x \sigma_n(x)$ and $x = 1$. Incidentally, we obtain a simpler expression for Bernoulli numbers in terms of Stirling numbers by setting $k = 1$ in (6.99):
\[
\sum_{k \geq 0} \binom{m}{k} (-1)^k \frac{k!}{k+1} = B_m
\]

6.75 Sam Loyd [204, pages 288 and 378] gave the construction

![Diagram](image)

and claimed to have invented (but not published) the $64 = 65$ arrangement in 1858. (Similar paradoxes go back at least to the eighteenth century, but Loyd found better ways to present them.)

6.76 We expect $A_m/A_{m-1} \approx \phi$, so we try $A_{m-1} = 618034 + r$ and $A_{m-2} = 381966 - r$. Then $A_{m-3} = 236068 + 2r$, etc., and we find $A_{m-18} = 144 - 2584r$, $A_{m-19} = 154 + 4181r$. Hence $r = 0$, $x = 154$, $y = 144$, $m = 20$.

6.77 If $P(F_{n+1}, F_n) = 0$ for infinitely many even values of $n$, then $P(x, y)$ is divisible by $U(x, y) = 1$, where $U(x, y) = x^2 - xy - y^2$. For if $t$ is the total degree of $P$, we can write
\[
P(x, y) = \sum_{k=0}^{t} q_k x^k y^{t-k} + \sum_{j+k=t} r_{j,k} x^i y^k = Q(x, y) + R(x, y)
\]
Then
\[
\frac{P(F_{n+1}, F_n)}{F_n^t} = \sum_{k=0}^{t} q_k \left( \frac{F_{n+1}}{F_n} \right)^k + O(1/F_n)
\]