slightest deviation from these paths causes the trajectory to depart from the critical point. Since none of the critical points is stable, one might expect that most trajectories will approach infinity for large $t$. However, it can be shown that all solutions remain bounded as $t \to \infty$; see Problem 5. In fact, it can be shown that all solutions ultimately approach a certain limiting set of points that has zero volume. Indeed, this is true not only for $r > r_2$ but for all positive values of $r$.

A plot of computed values of $x$ versus $t$ for a typical solution with $r > r_2$ is shown in Figure 9.8.2. Note that the solution oscillates back and forth between positive and negative values in a rather erratic manner. Indeed, the graph of $x$ versus $t$ resembles a random vibration, although the Lorenz equations are entirely deterministic and the solution is completely determined by the initial conditions. Nevertheless, the solution also exhibits a certain regularity in that the frequency and amplitude of the oscillations are essentially constant in time.

The solutions of the Lorenz equations are also extremely sensitive to perturbations in the initial conditions. Figure 9.8.3 shows the graphs of computed values of $x$ versus $t$