for the two solutions whose initial points are (5, 5, 5) and (5.01, 5, 5). The dashed graph is the same as the one in Figure 9.8.2, while the solid graph starts at a nearby point. The two solutions remain close until \( t \) is near 10, after which they are quite different and, indeed, seem to have no relation to each other. It was this property that particularly attracted the attention of Lorenz in his original study of these equations, and caused him to conclude that detailed long-range weather predictions are probably not possible.

The attracting set in this case, although of zero volume, has a rather complicated structure and is called a **strange attractor**. The term **chaotic** has come into general use to describe solutions such as those shown in Figures 9.8.2 and 9.8.3.

To determine how and when the strange attractor is created it is illuminating to investigate solutions for smaller values of \( r \). For \( r = 21 \) solutions starting at three different initial points are shown in Figure 9.8.4. For the initial point (3, 8, 0) the solution begins to converge to the point \( P_3 \) almost at once; see Figure 9.8.4a. For the second initial point (5, 5, 5) there is a fairly short interval of transient behavior, after which the solution converges to \( P_2 \); see Figure 9.8.4b. However, as shown in Figure 9.8.4c, for the third initial point (5, 5, 10) there is a much longer interval of transient chaotic behavior before the solution eventually converges to \( P_2 \). As \( r \) increases, the duration of the chaotic transient behavior also increases. When \( r = r_3 \approx 24.06 \), the chaotic transients appear to continue indefinitely and the strange attractor comes into being.

One can also show the trajectories of the Lorenz equations in the three-dimensional phase space, or at least projections of them in various planes. Figures 9.8.5 and 9.8.6

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**FIGURE 9.8.4** Plots of \( x \) versus \( t \) for three solutions of Lorenz equations with \( r = 21 \). (a) Initial point is (3, 8, 0). (b) Initial point is (5, 5, 5). (c) Initial point is (5, 5, 10).