In many important physical problems there are two or more independent variables, so that the corresponding mathematical models involve partial, rather than ordinary, differential equations. This chapter treats one important method for solving partial differential equations, a method known as separation of variables. Its essential feature is the replacement of the partial differential equation by a set of ordinary differential equations, which must be solved subject to given initial or boundary conditions. The first section of this chapter deals with some basic properties of boundary value problems for ordinary differential equations. The desired solution of the partial differential equation is then expressed as a sum, usually an infinite series, formed from solutions of the ordinary differential equations. In many cases we ultimately need to deal with a series of sines and/or cosines, so part of the chapter is devoted to a discussion of such series, which are known as Fourier series. With the necessary mathematical background in place, we then illustrate the use of separation of variables on a variety of problems arising from heat conduction, wave propagation, and potential theory.

10.1 Two-Point Boundary Value Problems

Up to this point in the book we have dealt with initial value problems, consisting of a differential equation together with suitable initial conditions at a given point. A typical example, which was discussed at length in Chapter 3, is the differential equation

\[ y'' + p(t)y' + q(t)y = g(t), \]  

(1)