8.24 (a) Any one of the dice ends up in J’s possession with probability \( p = \frac{1}{6} + \left(\frac{5}{6}\right)^2 \), hence \( p = \frac{6}{11} \). Let \( q = \frac{5}{11} \). Then the pgf for J’s total holdings is \( (q + pz)^{2n+1} \) with mean \((2n+1)p\) and variance \((2n+1)pq\), by (8.61).
(b) \( \left(\frac{1}{2}\right)p^3q^2 + \left(\frac{1}{2}\right)p^4q + \left(\frac{1}{2}\right)p^5 = \frac{8176}{161051} \approx .585 \).

8.25 The pgf for the current stake after \( n \) rolls is \( G_n(z) \), where
\[
G_0(z) = z^A; \\
G(z) = \sum_{k=1}^{6} G_n (z^{2^{(k-1)/5}})/6, \quad \text{for } n > 0.
\]
(The noninteger exponents cause no trouble.) It follows that \( \text{Mean}(G_n) = \text{Mean}(G_{n-1}) \), and \( \text{Var}(G_n) + \text{Mean}(G_n)^2 = \frac{9}{16} (\text{Var}(G_{n-1}) + \text{Mean}(G_{n-1})^2) \).
So the mean is always \( A \), but the variance grows to \( \left(\left(\frac{9}{16}\right)^n - 1\right)A^2 \).

8.26 The pgf \( F_{1,n}(z) \) satisfies \( F_{1,n}'(z) = F_{1,n}'(1) \); hence \( \text{Mean}(F_{1,n}) = F_{1,n}'(1) = \frac{[n > 1]}{1} \) and \( F_{1,n}''(1) = \frac{[n > 2]}{1} \); the variance is easily computed. (In fact, we have
\[
F_{1,n}(z) = \sum_{0 < k < n / l} \frac{1}{k!} \left( \frac{z-1}{l} \right)^k,
\]
which approaches a Poisson distribution with mean \( 1/l \) as \( n \to \infty \).

8.27 \( \left( n^2\Sigma_3 - 3n\Sigma_2\Sigma_1 + 2\Sigma_3^2 \right)/n(n-1)(n-2) \) has the desired mean, where \( \Sigma_k = X_1^k + \cdots + X_n^k \). This follows from the identities
\[
E\Sigma_3 = n\mu_3; \\
E(\Sigma_2\Sigma_1) = n\mu_3 + n(n-1)\mu_2\mu_1; \\
E(\Sigma_3^2) = n\mu_3 + 3n(n-1)\mu_2\mu_1 + n(n-1)(n-2)\mu_3.
\]
Incidentally, the third cumulant is \( \kappa_3 = E( (X-EX)^3 ) \), but the fourth cumulant does not have such a simple expression; we have \( \kappa_4 = E ( (X-EX)^4 ) - 3( VX)^2 \).

8.28 (The exercise implicitly calls for \( p = q = \frac{1}{2} \), but the general answer is given here for completeness.) Replace \( H \) by \( pz \) and \( T \) by \( qz \), getting \( S_A(z) = p^2q^2z/\left(1 - pz\right)^2 \) and \( S_B(z) = pq^2z^2/\left(1 - qz\right)^2 \). The pgf for the conditional probability that Alice wins at the nth flip, given that she wins the game, is
\[
\frac{S_A(z)}{S_A(1)} = \frac{3 + q + p + 2pq}{1-pz} \frac{1-pq}{1-qz} \frac{1-pq}{1-pqz^2}
\]
This is a product of pseudo-pgf’s, whose mean is \( 3 + p/q + q/p + 2pq/(1 - pq) \). The formulas for Bill are the same but without the factor \( q/(1 - pz) \), so Bill’s mean is \( 3 + q/p + 2pq/(1 - pq) \). When \( p = q = \frac{1}{2} \), the answer in case (a) is...