to 1000 as above. Results in Fig. 2 show manipulation probability and expected regret as we vary $\phi$ and consider desired alternatives 1, 2 and 3.\footnote{Under IC (i.e., when $\phi = 1$), alternatives are probabilistically indistinguishable, so we show one row only.} While manipulation probability is high for these near-top alternatives (when $n$ is small), expected regret (normalized in percentage terms) is negligible, with a maximum of 4%, and then only when the distribution is close to impartial culture ($\phi = 0.8$) and $n = 100$ (nearly 10% manipulators). As above, when manipulators want $d = 2$, expected regret is highest. Of some interest is the connection to both theoretical and empirical work that shows phase transitions often occur when the number of manipulators is roughly the square root of the number of sincere voters: any less makes manipulation very unlikely, while any more makes manipulation likely. While most of this work analyzes complete information settings, our results above show that with realistic preference distributions—even with restricted knowledge on the part of manipulators—the probability of manipulation is sometimes quite significant with far fewer manipulators than suggested by past work. Despite this, expected regret remains relatively small.

Fig. 3 shows the average and maximum running times of the MIP required to compute the optimal manipulation for the problems described above. As can be seen, even with a large number of sampled profiles, the MIP can be solved quickly across a range of problem sizes and distributions.

7 CONCLUDING REMARKS

Our primary contribution is an empirical framework for the computation of optimal manipulation strategies when manipulators have incomplete information about voter preferences. This is an important methodology for the analysis of the manipulability of voting rules in realistic circumstances, without the need to restrict the analysis to specific voting rules or priors. Our experiments indicate that our algorithms are quite tractable. Furthermore, our results suggest that manipulation may not be as serious a problem as is commonly believed when realistic informational models are used, or when the quality of the outcome, rather than societal justice, is the main objective. Our empirical results, which exploit several innovations introduced in this paper, demonstrate this in the case of Borda voting; but our approach is easily adapted to different types of manipulation under different scoring rules, and can be applied to any “utility-based” voting rule with appropriate formulation of the optimization problem. Thus, our approach provides a compelling framework for the comparison of voting rules.

One nonstandard aspect of our approach is the use of the score under a voting rule as a proxy for social welfare. A similar regret-based approach is taken in preference elicitation [22]; and it is implicit in work on approximating voting rules [28, 5], which assumes that approximating the score also gives an approximation to the desirability of an alternative. One strong argument in favor of this view is that scores of certain voting rules, such as Borda, are provably good proxies for utilitarian social welfare when utilities are drawn from specific distributions [37]. That said, our framework can be used, in principle, to analyze any measure of impact or loss.

Our work suggests a number of interesting future directions. Of course, we must study additional voting rules, social welfare measures, and manipulator objectives within our framework to further demonstrate its viability. While our results suggest that incomplete knowledge limits the ability of a manipulating coalition to impact the results of an election, our framework can also be used to directly study the relationship between the “amount of information” (e.g., using entropy or equivalent sample size metrics) and the probability of manipulation. Finally, interesting computational questions arise within our approach: e.g., deriving tighter complexity results for optimal manipulation given a collection of vote profiles; or deriving simpler classes of manipulation policies (e.g., uncertainty-sensitive variants of the balanced manipulation strategy) that can be more readily optimized by a manipulating coalition.