8.32 By symmetry, we can reduce each month’s situation to one of four possibilities:

D, the states are diagonally opposite;
A, the states are adjacent and not Kansas;
K, the states are Kansas and one other;
S, the states are the same.

Considering the Markovian transitions, we get four equations:

\[
\begin{align*}
D &= 1 + z(\frac{5}{9}D + \frac{4}{9}K) \\
A &= z(\frac{5}{9}A + \frac{4}{9}K) \\
K &= z(\frac{5}{9}D + \frac{4}{9}A + \frac{4}{12}K) \\
S &= z(\frac{5}{9}D + \frac{4}{9}A + \frac{4}{12}K)
\end{align*}
\]

whose sum is \(D + K + A + S = 1 + z(D + A + K)\). The solution is

\[
S = \frac{81z - 45z^2 - 423}{243 - 243z + 24z^2 + 8z^3}
\]

but the simplest way to find the mean and variance may be to write \(z = 1 + w\) and expand in powers of \(w\), ignoring multiples of \(w^2\):

\[
\begin{align*}
D &= \frac{27}{16} + \frac{1593}{512}w + \cdots \\
A &= \frac{9}{8} + \frac{2115}{256}w + \cdots \\
K &= \frac{15}{8} + \frac{2661}{256}w + \cdots
\end{align*}
\]

Now \(S'(1) = \frac{27}{16} + \frac{9}{8} + \frac{15}{8} = \frac{75}{16}\), and \(\frac{1}{2}S''(1) = \frac{1593}{512} + \frac{2115}{256} + \frac{2661}{256} = \frac{11145}{512}\). The mean is \(\frac{75}{16}\) and the variance is \(\frac{105}{4}\). (Is there a simpler way?)

8.33 First answer: Clearly yes, because the hash values \(h_1, \ldots, h_n\) are independent. Second answer: Certainly no, even though the hash values \(h_1, \ldots, h_n\) are independent. We have \(Pr(X_1 = 0) = \sum_{k=1}^{n} s_k (\frac{(m-1)}{m}) = (1 - s_1)/m\), but \(Pr(X_1 = X_2 = 0) = \sum_{k=1}^{n} s_k (\text{if } k > 2) (\frac{(m-1)}{m})^2 = (1 - s_1 - s_2) (\frac{(m-1)}{m})^2 \neq Pr(X_1 = 0) Pr(X_2 = 0)\).

8.34 Let \([z^m] S(z)\) be the probability that Gina has advanced \(< m\) steps after taking \(n\) turns. Then \(S_m(1)\) is her average score on a par-\(m\) hole; \([z^n] S(z)\) is the probability that she loses such a hole against a steady player; and \(1 - [z^{m-1}] S(z)\) is the probability that she wins it. We have the recurrence

\[
\begin{align*}
S_0(z) &= 0; \\
S_m(z) &= (1 + pz S_{m-2}(z) + qz S_{m-1}(z)) / (1 - rz), \quad \text{for } m > 0.
\end{align*}
\]