To solve part (a), it suffices to compute the coefficients for \( m, n \leq 4 \); it is convenient to replace \( z \) by \( 100w \) so that the computations involve nothing but integers. We obtain the following tableau of coefficients:

\[
\begin{array}{cccccc}
S_0 & 0 & 0 & 0 & 0 & 0 \\
S_1 & 1 & 4 & 16 & 64 & 256 \\
S_2 & 1 & 95 & 744 & 4432 & 23552 \\
S_3 & 1 & 100 & 9065 & 104044 & 819080 \\
S_4 & 1 & 100 & 9975 & 868535 & 12964304 \\
\end{array}
\]

Therefore Gina wins with probability \( 1 - .868535 = .131465 \); she loses with probability \( .12964304 \). (b) To find the mean number of strokes, we compute

\[
S_1(1) = \frac{25}{24}; \quad S_2(1) = \frac{4675}{2304}; \quad S_3(1) = \frac{667825}{221184}; \quad S_4(1) = \frac{85134475}{21233664}.
\]

(Incidentally, \( S_5(1) \approx 4.9995 \); she wins with respect to both holes and strokes on a par-5 hole, but loses either way when par is 3.)

8.35 The condition will be true for all \( n \) if and only if it is true for \( n = 1 \), by the Chinese remainder theorem. One necessary and sufficient condition is the polynomial identity

\[
(p_2+p_4+p_6+(p_1+p_3+p_5)w)(p_3+p_6+(p_1+p_4)z+(p_2+p_5)z^2)
\]

\[
= (p_1wz+p_2z^2+p_3w+z+p_5wz^2+p_6),
\]

but that just more-or-less restates the problem. A simpler characterization is

\[
(p_2+p_4+p_6)(p_3+p_6) = p_5, \quad (p_1+p_3+p_5)(p_2+p_5) = p_5,
\]

which checks only two of the coefficients in the former product. The general solution has three degrees of freedom: Let \( a_0 = a_1 = b_0 = b_1 = b_2 = 1 \), and put \( a = a_1b_1, \ p_2 = a_0b_2, \ p_3 = a_1b_0, \ p_4 = a_0b_1, \ p_5 = a_1b_1, \ p_6 = a_0b_0. \)

8.36 (a) \( \square \square \square \square \square \square \square \). (b) If the \( k \)th die has faces with \( s_1, \ldots, s_6 \) spots, let \( p_k(z) = z^{s_1} + \cdots + z^{s_6} \). We want to find such polynomials with

\[
p_1(z) \cdots p_n(z) = (z + z^2 + z^3 + z^4 + z^5 + z^6)^n.
\]

The irreducible factors of this polynomial with rational coefficients are \( z^n(z+1)^n(z^2+z+1)^n(z^2-z+1)^n \), hence \( p_k(z) \) must be of the form \( z^{d_k} (z+1)^{k_1} (z^2+z+1)^{k_2} (z^2-z+1)^{k_3} \). We must have \( d_k \geq 1 \), since \( p_k(0) = 0 \); and in fact \( d_k = 1 \), since \( a_1 \cdots + a_n = n \). Furthermore the condition \( p_k(1) = 0 \) implies that \( b_k = c_k = 1 \). It is now easy to see that \( 0 \leq d_5 \leq 2 \), since \( d_5 > 2 \) gives negative coefficients. When \( d = 0 \) and \( d = 2 \), we get the two dice in part (a); therefore the only solutions have \( k \) pairs of dice as in (a), plus \( n \) 2k ordinary dice, for some \( k \leq \frac{1}{2} n \).