over TCP ontologies with cMLNs that leverages the properties of equivalence classes and is guaranteed to inspect worlds in decreasing order of probability. We provide an upper bound on the error that is incurred by this algorithm based on the number of worlds and equivalence classes inspected, and a criterion to decide when results are guaranteed to be correct.

The rest of this paper is organized as follows. Section 2 describes the preliminaries on the DL $\mathcal{EL}^{++}$ and MLNs. Section 3 presents tightly coupled probabilistic DLs, and a complexity result showing the intractability of computing probabilities of atoms. In Section 4, we present conjunctive MLNs, and we analyze how their structure can be leveraged in the definition of well-behaved equivalence classes over possible worlds, as well as in an anytime algorithm that exploits this equivalence class approach in order to tractably compute a heuristic answer to the ranking of atoms according to their probabilities. Finally, Sections 5 and 6 discuss related work and conclusions, respectively.

2 Preliminaries

In this section, we briefly recall the description logic (DL) $\mathcal{EL}^{++}$ and Markov logic networks (MLNs).

2.1 The DL $\mathcal{EL}^{++}$

We now recall the syntax and the semantics of $\mathcal{EL}^{++}$, a tractable DL especially suited for representing large amounts of data. Intuitively, DLs model a domain of interest in terms of concepts and roles, which represent classes of individuals and binary relations between individuals, respectively. While we restrict ourselves to $\mathcal{EL}^{++}$ here, the general approach continues to be valid for any DL or ontology language for which instance checking is data-tractable (for instance, the closely related Datalog+- family of ontology languages contains such tractable subsets [3]). However, note that all results in this paper were derived for $\mathcal{EL}^{++}$, and thus certain results may not hold for other logics.

**Syntax and Semantics.** We first define concepts and then know ledge bases and instance checking in $\mathcal{EL}^{++}$. We assume pairwise disjoint sets $\mathbf{A}$, $\mathbf{R}$, and $\mathbf{I}$ of atomic concept names, role names, and individual names, respectively. Concepts are defined inductively via the construction shown in the first five rows of the Table in Figure 2; this table adopts the usual conventions of using $C$ and $D$ to refer to concepts, $r$ to refer to a role, and $a$ and $b$ to refer to individuals. The semantics of these concepts is given, as usual in first-order logics, in terms of an interpretation $I = (\Delta^I, \cdot^I)$. The domain $\Delta^I$ comprises a non-empty set of individuals and the interpretation function $\cdot^I$ maps each concept name $A \in \mathbf{A}$ to $A^I \subseteq \Delta^I$, each role name $r \in \mathbf{R}$ to a binary relation $r^I$ over $\Delta^I \times \Delta^I$, and each individual name $a \in \mathbf{I}$ to an individual $a^I \in \Delta^I$. The ex-