Intuitively, a probabilistic annotation $\lambda$ is used to describe the class of events in which the random variables in an MLN are compatible with the settings of the random variables described by $\lambda$, i.e., each $X_i$ has the value $x_i$.

**Definition 2.** Let $F$ be the FOL translation of an $\mathcal{EL}^\ast$ axiom, and $\lambda$ be a probabilistic annotation; a probabilistic $\mathcal{EL}^\ast$ axiom is of the form $F : \lambda$. We also refer to probabilistic axioms as annotated formulas.

Essentially, probabilistic axioms hold whenever the events associated with their annotations occur. Note that whenever a random variable’s value is left unspecified in an annotation, the variable is unconstrained; in particular, an empty annotation means that the formula holds in every possible world (we sometimes refer to these axioms as crisp).

**Definition 3.** Let $O$ be a set of (FOL translations of) probabilistic $\mathcal{EL}^\ast$ axioms and $M$ be an MLN. A tightly coupled probabilistic $\mathcal{EL}^\ast$ ontology (TCP ontology, or knowledge base) is of the form $KB = (O, M)$, where the probabilistic annotations of formulas in $O$ are relative to $M$.

Recall that random variables in our MLN setting are Boolean and written in the form of atoms over $R_{MLN}, V_{MLN}$, and $\Delta_{MLN}$; if $a$ is such an atom, $a = 1$ (resp., $a = 0$) denotes that the variable is true (resp., false); we also use the notation $\neg a$, respectively.

**Definition 4.** Let $KB = (O, M)$ be a probabilistic $\mathcal{EL}^\ast$ ontology, and $\lambda$ be a possible world. The (non-probabilistic) $\mathcal{EL}^\ast$ ontology induced from $KB$ by $\lambda$, denoted $O_\lambda$, is the set $\{F_i \mid F_i : \lambda_i \in O \text{ and } \theta_i, \lambda_i \subseteq \lambda\}$, where $\theta_i$ is an mgu for $\lambda$ and $\lambda_i$.

The annotation of ontological axioms offers a clear modeling advantage by enabling a clear separation of concerns between the task of ontological modeling and the task of modeling the uncertainty around the axioms in the ontology. More precisely, in our formalism, it is possible to express the fact that the probabilistic nature of an ontological axiom is determined by elements that are outside of the domain modeled by the ontology.

**Example 3.** Consider the form-labeling ontology of Example 1 and the MLN of Example 2. In general, we expect only certain axioms to be probabilistic, while others are necessarily crisp. In our case, the fact that fields and text blocks are disjoint sets of objects, and that fields are labeled by text blocks are considered crisp axioms, since we do not want these assumptions to be violated by any model. However, we want to accept models where some field is left unlabeled, and therefore violating the axiom $Field \subseteq \exists label. Text$ is possible. In addition, we want to link the probability that this axiom holds to the heuristics used to produce the actual labeling of the field, which are out of the domain of the form-labeling ontology. The following is a possible probabilistic $\mathcal{EL}^\ast$ ontology modeling this setup, where the first-order representation of the $\mathcal{EL}^\ast$ axioms discussed above is used to explicitly state the relationships between variables and constants in the MLN and in the ontology.

\[
\forall X. field(X) \rightarrow \exists Y. label(X, Y) \land text(Y) : \left\{ (\text{canLabel}(Y, X), 1) \right\};
\]

\[
\forall X. label(X, Y) \rightarrow field(X) : \{\};
\]

\[
\forall X. label(X, Y) \land text(Y) \rightarrow \perp : \{\}.
\]

**Data Complexity.** In this setting, we extend the usual concept of data complexity as follows: the set of formulas in the MLN are considered to be fixed, as is the TBox; on the other hand, the sets $I$ and $\Delta_{MLN}$ are not, and therefore the ground ABox on the ontology side and the set of random variables on the MLN side are not fixed, either.

### 3.2 Semantics

The semantics of TCP ontologies is given relative to probabilistic distributions over interpretations of the form $I_{MLN} = \langle D, w \rangle$, where $D$ is a database over $I \cup \Delta_N$, and $w$ is a world. We usually abbreviate “true : $\lambda$” with “$\lambda$”.

**Definition 5.** An interpretation $I_{MLN} = \langle D, w \rangle$ satisfies an annotated formula $F : \lambda$, denoted $I_{MLN} \models F : \lambda$, iff whenever there exists an mgu $\theta$ such that for all $\langle V_i, x_i \rangle \in \lambda$ it holds that $X_i = \theta V_i$ and $w[i] = x_i$, then $D \models \theta F$.

A probabilistic interpretation is then a probability distribution $Pr$ over the set of all possible interpretations such that only a finite number of interpretations are mapped to a non-zero value. The probability of an annotated formula $F : \lambda$, denoted $Pr(F : \lambda)$, is the sum of all $Pr(I_{MLN})$ such that $I_{MLN}$ satisfies $F : \lambda$.

**Definition 6.** Let $Pr$ be a probabilistic interpretation, and $F : \lambda$ be an annotated formula. We say that $Pr$ satisfies (or is a model of) $F : \lambda$ iff $Pr(F : \lambda) = 1$. Furthermore, $Pr$ is a model of a probabilistic $\mathcal{EL}^\ast$ ontology $KB = (O, M)$ iff: (i) $Pr$ satisfies all annotated formulas in $O$, and (ii) $1 - Pr(\text{false} : \lambda) = Pr_M(\lambda)$ for all possible worlds $\lambda$, where $Pr_M(\lambda)$ is the probability of $\bigwedge_{(V_i, x_i) \in \lambda}(V_i = x_i)$ in the MLN $M$ (and computed in the same way as $P(X = x)$ in Section 2.2).

In Definition 6 above, condition (ii) is stating that the probability values that $Pr$ assigns are in accordance with those of $MLN$ $M$ (note that the equality in the definition implies that $Pr(\text{true} : \lambda) = Pr_M(\lambda)$) and that they are adequately distributed (since $Pr(\text{true} : \lambda) + Pr(\text{false} : \lambda) = 1$).

In the following, we are interested in computing the probabilities of atoms in a TCP ontology, working towards ranking of entailed atoms based on their probabilities.

**Definition 7.** Let $KB = (O, M)$ be a TCP ontology, and $a$ be a ground atom that is constructed from predicates and