14.7 The Markov blanket of a variable is defined on page 517. Prove that a variable is independent of all other variables in the network, given its Markov blanket and derive Equation (14.12) (page 538).
Exercises 561

14.8  Consider the network for car diagnosis shown in Figure 14.21.

a. Extend the network with the Boolean variables *IceWeather* and *StarterMotor*.

b. Give reasonable conditional probability tables for all the nodes.

c. How many independent values are contained in the joint probability distribution for eight Boolean nodes, assuming that no conditional independence relations are known to hold among them?

d. How many independent probability values do your network tables contain?

e. The conditional distribution for *Starts* could be described as a noisy-AND distribution. Define this family in general and relate it to the noisy-OR distribution.

14.9  Consider the family of linear Gaussian networks, as defined on page 520.

a. In a two-variable network, let $X_1$ be the parent of $X_2$, let $X_i$ have a Gaussian prior, and let $P(X_2 | X_i)$ be a linear Gaussian distribution. Show that the joint distribution $P(X_1, X_2)$ is a multivariate Gaussian, and calculate its covariance matrix.

b. Prove by induction that the joint distribution for a general linear Gaussian network on $X_1, \ldots, X_n$ is also a multivariate Gaussian.

14.10  The probit distribution defined on page 522 describes the probability distribution for a Boolean child, given a single continuous parent.

a. How might the definition be extended to cover multiple continuous parents?

b. How might it be extended to handle a *multivalued* child variable? Consider both cases where the child’s values are ordered (as in selecting a gear while driving, depending on speed, slope, desired acceleration, etc.) and cases where they are unordered (as in selecting bus, train, or car to get to work). (Hint: Consider ways to divide the possible values into two sets, to mimic a Boolean variable.)

14.11  In your local nuclear power station, there is an alarm that senses when a temperature gauge exceeds a given threshold. The gauge measures the temperature of the core. Consider the Boolean variables $A$ (alarm sounds), $FA$ (alarm is faulty), and $Fr$ (gauge is faulty) and the multivalued nodes $G$ (gauge reacting) and $T$ (actual core temperature).

a. Draw a Bayesian network for this domain, given that the gauge is more likely to fail when the core temperature gets too high.

b. Is your network a polytree? Why or why not?

c. Suppose there are just two possible actual and measured temperatures, normal and high; the probability that the gauge gives the correct temperature is $x$ when it is working, but when it is faulty. Give the conditional probability table associated with $G$.

d. Suppose the alarm works correctly unless it is faulty, in which case it never sounds. Give the conditional probability table associated with $A$.

e. Suppose the alarm and gauge are working and the alarm sounds. Calculate an expression for the probability that the temperature of the core is low, high, in terms of the various conditional probabilities in the network.