We can now derive the recurrence
\[ D''(1) = (n-11)D''(1)/(n+1) + (8n-2)/7, \]
which has the solution \( \frac{2}{63}(n+2)(26n+15) \) for all \( n \geq 11 \) (regardless of initial conditions). Hence the variance comes to \( \frac{12}{49}(n+2)(212n+123) \) for \( n \geq 11 \).

8.62 (Another question asks if a given sequence of purported cumulants comes from any distribution whatever; for example, \( \kappa_2 \) must be nonnegative, and \( \kappa_4 + 3\kappa_2^2 = E((X-\mu)^4) \) must be at least \( (E((X-\mu)^2))^2 = \kappa_2^2 \), etc. A necessary and sufficient condition for this other problem was found by Hamburger \[6, 144].)

8.63 (Another question asks if there is a simple rule to tell whether \( H \) or \( T \) is preferable.) Conway conjectures that no such ties exist, and moreover that there is only one cycle in the directed graph on \( 2^l \) vertices that has an arc from each sequence to its “best beater!”

9.1 True if the functions are all positive. But otherwise we might have, say, \( f_1(n) = n^3 + n^2, f_2(n) = -n^3, g_1(n) = n^4 + n, g_2(n) = -n^4 \).

9.2 (a) We have \( n^{\ln n} \ll \exp(n) \ll \ln n \), since \( (\ln n)^2 \ll n \ln n \ll n \ln \ln n \).
(b) \( n^{\ln \ln n} \ll (\ln n)! \ll n^{\ln n} \).
(c) Take logarithms to show that \( (n!)! \) wins.
(d) \( n^{\ln \ln n} \ll \phi^{\ln n} = n^{2\ln \phi}; H_{r_n} \sim n \ln \phi \) wins because \( \phi^2 = \phi + 1 < e \).

9.3 Replacing \( kn \) by \( 0(n) \) requires a different \( C \) for each \( k \); but each \( 0 \) stands for a single \( C \). In fact, the context of this 0 requires it to stand for a set of functions of two variables \( k \) and \( n \). It would be correct to write \( \sum_{k=1}^{n} kn = \sum_{k=1}^{n} O(n^2) = O(n^3) \).

9.4 For example, \( \lim_{n \to \infty} O(1/n) = 0 \). On the left, \( O(1/n) \) is the set of all functions \( f(n) \) such that there are constants \( C \) and \( n_0 \) with \( |f(n)| \leq C/n \) for all \( n \geq n_0 \). The limit of all functions in that set is 0, so the left-hand side is the singleton set \( \{0\} \). On the right, there are no variables; 0 represents \( \{0\} \), the (singleton) set of all “functions of no variables, whose value is zero!” (Can you see the inherent logic here? If not, come back to it next year; you probably can still manipulate \( O \)-notation even if you can’t shape your intuitions into rigorous formalisms.)

9.5 Let \( f(n) = n^n \) and \( g(n) = 1 \); then \( n \) is in the left set but not in the right, so the statement is false.

9.6 \( n \ln n + \gamma n + O(\sqrt{n} \ln n) \).

9.7 \( (1 - e^{-1/n})^{-1} = nB_0 - B_1 + B_2 n^{-1}/2! + \cdots = n + \frac{1}{2} + O(n^{-1}) \).

9.8 For example, let \( f(n) = [n/2]^2 + n, g(n) = \left( \left[ \frac{n}{2} \right] - 1! \right) \left[ \frac{n}{2} \right]! + n \). These functions, incidentally, satisfy \( f(n) = O(g(n)) \) and \( g(n) = O(n f(n)) \); more extreme examples are clearly possible.