9.9 (For completeness, we assume that there is a side condition \( n \to \infty \), so that two constants are implied by each 0.) Every function on the left has the form \( a(n) + b(n) \), where there exist constants \( m_0, B, no, C \) such that 
\[
|a(n)| \leq B|f(n)| \quad \text{for} \quad n \geq m_0 \quad \text{and} \quad |b(n)| \leq C|g(n)| \quad \text{for} \quad n \geq no.
\]
Therefore the left-hand function is at most \( \max(B, C)(|f(n)| + |g(n)|) \), for \( n \geq \max(m_0, no) \), so it is a member of the right side.

9.10 If \( g(x) \) belongs to the left, so that \( g(x) = \cos y \) for some \( y \), where 
\[
|y| \leq C|x| \quad \text{for} \quad \text{some} \quad C,
\]
then \( 0 \leq 1 - g(x) = 2 \sin^2(y/2) \leq \frac{1}{2} y^2 \leq \frac{1}{2} C^2 x^2; \) hence the set on the left is contained in the set on the right, and the formula is true.

9.11 The proposition is true. For if, say, \( |x| \leq |y| \), we have \( (x + y)^2 \leq 4y^2 \). Thus \( (x + y)^2 = O(x^2) + O(y^2) \). Thus \( O(x+y)^2 = O((x+y)^2) = O(O(x^2) + O(y^2)) = O(x^2) + O(y^2) \).

9.12 \( 1 + 2n + O(n^{-2}) = (1 + 2/n)(1 + O(n^{-2})/(1 + 2/n)) \) by (9.26), and 
\( 1/(1 + 2/n) = O(1) \); now use (9.26).

9.13 \( n^{n}(1 + 2n^{-1} + O(n^{-2}))^{n} = n^{n}\exp(n(2n^{-1} + O(n^{-2}))) = e^2 n^{n} + O(n^{n-1}) \).

9.14 It is \( n^{n+\beta} \exp((n + \beta)(\alpha/n - \frac{1}{2} \alpha^2/n^2 + O(n^{-3})) \)

9.15 \( \ln \left( \frac{3n}{\ln n} \right) = 3\ln 3 - \ln n + \frac{1}{2} \ln 3 - \ln 2\pi + \left( \frac{1}{3\sigma} - \frac{1}{4} \right)n^{-1} + O(n^{-3}) \), so the answer is
\[
\frac{3^{3n+1/2}}{2\pi n}\left(1 - \frac{3}{8} n^{-1} + \frac{3}{8} n^{-2} + O(n^{-3}) \right).
\]

9.16 If \( l \) is any integer in the range \( a \leq l < b \) we have
\[
\int_{l}^{l+1} B(x)f(l+x) \, dx = \int_{l}^{l+1/2} B(x)f(l+x) \, dx - \int_{l}^{l+1} B(l-x)f(l+x) \, dx
\]
\[
= \int_{l+1/2}^{l+1} B(x)(f(l+x) - f(l+1-x)) \, dx.
\]
Since \( l + x \geq l + 1 - x \) when \( x \geq \frac{1}{2} \), this integral is positive when \( f(x) \) is nondecreasing.

9.17 \( \sum_{m \geq 0} B_m(\frac{1}{2})z^m/m! = ze^{z/2}/(e^z - 1) = z/(e^{z/2} - 1) - z/(e^z - 1) \)

9.18 The text's derivation for the case \( \alpha = 1 \) generalizes to give
\[
b_k(n) = 2^{(2n+1/2)\alpha} \frac{(2n)\alpha!}{(2m)_\alpha} e^{-k^2\alpha/n}, \quad c_k(n) = 2^{2n\alpha} n^{-1/2+3\epsilon} e^{-k^2\alpha/n},
\]
the answer is \( 2^{2n\alpha}(\pi n)^{(1-\alpha)/2}\alpha^{-1/2}(1 + O(n^{-1/2+3\epsilon})) \).