This relation between a function \( f \) and its Fourier coefficients is known as Parseval’s equation. Parseval’s equation is very important in the theory of Fourier series and is discussed further in Section 11.6.

**Hint:** Multiply Eq. (i) by \( f(x) \), integrate from \(-L\) to \(L\), and use the Euler–Fourier formulas.

18. This problem indicates a proof of convergence of a Fourier series under conditions more restrictive than those in Theorem 10.3.1.

(a) If \( f \) and \( f' \) are piecewise continuous on \(-L \leq x < L\), and \( f \) is periodic with period \(2L\), show that \( n a_n \) and \( n b_n \) are bounded as \( n \to \infty \).

**Hint:** Use integration by parts.

(b) If \( f \) is continuous on \(-L \leq x \leq L\) and periodic with period \(2L\), and if \( f' \) and \( f'' \) are piecewise continuous on \(-L \leq x < L\), show that \( n^2 a_n \) and \( n^2 b_n \) are bounded as \( n \to \infty \).

Use this fact to show that the Fourier series for \( f \) converges at each point in \(-L \leq x \leq L\).

Why must \( f \) be continuous on the closed interval?

**Hint:** Again, use integration by parts.

**10.4 Even and Odd Functions**

Before looking at further examples of Fourier series it is useful to distinguish two classes of functions for which the Euler–Fourier formulas can be simplified. These are even and odd functions, which are characterized geometrically by the property of symmetry with respect to the \( y \)-axis and the origin, respectively (see [Figure 10.4.1](#)).

Analytically, \( f \) is an **even function** if its domain contains the point \(-x\) whenever it contains the point \(x\), and if

\[
f(-x) = f(x)
\]