3. Define a function \( k \) of period \( 2L \) so that

\[
k(x) = f(x), \quad 0 \leq x \leq L,
\]

and let \( k(x) \) be defined for \((-L, 0)\) in any way consistent with the conditions of Theorem 10.3.1. Sometimes it is convenient to define \( k(x) \) to be zero for \(-L < x < 0\). The Fourier series for \( k \), which involves both sine and cosine terms, also represents \( f \) on \([0, L]\), regardless of the manner in which \( k(x) \) is defined in \((-L, 0)\). Thus there are infinitely many such series, all of which converge to \( f(x) \) in the original interval.

Usually, the form of the expansion to be used will be dictated (or at least suggested) by the purpose for which it is needed. However, if there is a choice as to the kind of Fourier series to be used, the selection can sometimes be based on the rapidity of convergence. For example, the cosine series for the triangular wave [Eq. (20) of Section 10.2] converges more rapidly than the sine series for the sawtooth wave [Eq. (9) in this section], although both converge to the same function for \( 0 \leq x < L \). This is due to the fact that the triangular wave is a smoother function than the sawtooth wave and is therefore easier to approximate. In general, the more continuous derivatives possessed by a function over the entire interval \(-\infty < x < \infty\), the faster its Fourier series will converge. See Problem 18 of Section 10.3.

**Example 2**

Suppose that

\[
f(x) = \begin{cases} 
1 - x, & 0 < x \leq 1, \\
0, & 1 < x \leq 2.
\end{cases}
\]

As indicated previously, we can represent \( f \) either by a cosine series or by a sine series. Sketch the graph of the sum of each of these series for \(-6 \leq x \leq 6\).

In this example \( L = 2 \), so the cosine series for \( f \) converges to the even periodic extension of \( f \) of period 4, whose graph is sketched in Figure 10.4.4.

Similarly, the sine series for \( f \) converges to the odd periodic extension of \( f \) of period 4. The graph of this function is shown in Figure 10.4.5.

**Figure 10.4.4** Even periodic extension of \( f(x) \) given by Eq. (13).

**Figure 10.4.5** Odd periodic extension of \( f(x) \) given by Eq. (13).