The steady-state temperature satisfies \( v''(x) = 0 \) and the boundary conditions \( v(0) = 20 \) and \( v(30) = 50 \). Thus \( v(x) = 20 + x \). The transient distribution \( w(x, t) \) satisfies the heat conduction equation

\[
w_{xx} = w_t, \tag{21}\]

the homogeneous boundary conditions

\[
w(0, t) = 0, \quad w(30, t) = 0, \tag{22}\]

and the modified initial condition

\[
w(x, 0) = 60 - 2x - (20 + x) = 40 - 3x. \tag{23}\]

Note that this problem is of the form (1), (2), (3) with \( f(x) = 40 - 3x \), \( \alpha^2 = 1 \), and \( L = 30 \). Thus the solution is given by Eqs. (4) and (6).

Figure 10.6.1 shows a plot of the initial temperature distribution \( 60 - 2x \), the final temperature distribution \( 20 + x \), and the temperature at two intermediate times found by solving Eqs. (21) through (23). Note that the intermediate temperature satisfies the boundary conditions (19) for any \( t > 0 \). As \( t \) increases, the effect of the boundary conditions gradually moves from the ends of the bar toward its center.

**FIGURE 10.6.1** Temperature distributions at several times for the heat conduction problem of Example 1.

**Bar with Insulated Ends.** A slightly different problem occurs if the ends of the bar are insulated so that there is no passage of heat through them. According to Eq. (2) in Appendix A, the rate of flow of heat across a cross section is proportional to the rate of change of temperature in the \( x \) direction. Thus in the case of no heat flow the boundary conditions are

\[
u_x(0, t) = 0, \quad u_x(L, t) = 0, \quad t > 0. \tag{24}\]

The problem posed by Eqs. (1), (3), and (24) can also be solved by the method of separation of variables. If we let

\[
u(x, t) = X(x)T(t), \tag{25}\]